# Fiscal Federalism and Lobbying

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#### Abstract

Which functions should be decentralized (resp. centralized) once lobbying behavior is taken into account? In a two-region economy, two regional firms may either lobby *in* the market, to increase regional public good provision, or *for* the (regional) market, to gain access to it. We prove that lobbying is less disruptive for social welfare under *decentralization* in the former case, and under *centralization* in the latter. When lobbying in the market, firms' interests are aligned (both gain from an increase in regional public good supply), hence decentralization makes coordination among local lobbies more difficult and more costly. On the contrary, when lobbying is in the market, firms' interests are conflicting (each one striving for monopoly power), hence centralization, by forcing more competition between lobbies, lowers the rent they can extract from policy makers. We use these results to briefly comment upon the (de)centralization process in Europe.

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JEL classification: D70, H23, H77

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# 1 Introduction

One of the most fundamental questions in the theory of fiscal federalism concerns the correct allocation of functions to different levels of government. This question has not only theoretical appeal. Given the recent and widespread tendency towards decentralization within countries, and centralization (of some functions) across countries, it also has a deep policy content. Economists are not completely devoid of answers. For example, according to Oates' (1972) celebrated decentralization theorem, we should centralize (decentralize) functions with more (less) spillover effects and less (more) heterogeneity of preferences across jurisdictions. In its simplicity, this is a recipe which can carry one some way (see for instance, Alesina, Angeloni, Schuknecht, 2001, on the European Union). However, an important limitation of Oates's analysis is that he assumes welfare maximizing governments, and it is not clear how far his insights could go in more realistic political environments.

Consider, for instance, the current debate on the role that European Union (EU) institutions should play in fields such as labor markets institutions, competition and regulation policy, education, pensions, infrastructures etc. In these fields, currently largely under the control of national governments, many observers would agree that the most important policy distortions come from the pressure of powerful organized interests' groups on governments (e.g. Tabellini and Wyplosz, 2004). The important policy question, over which theoretical analysis should attempt to cast some light, is then whether these pressures are likely to become more or less powerful once these functions were centralized at the EU level. However, this is not the issue that has been considered in the attempts to extend Oates' analysis to a political economy framework (Besley and Coate, 2002; Lockwood, 2002). Moreover, despite the large economic literature on lobbying (e.g. Grossman and Helpman, 2001), very few studies have concentrated on the relationship between interest groups and decentralization. And when they have done so, they only focussed on the higher heterogeneity of preferences under centralization as the main discriminating factor (e.g. De Melo et al., 1993, and Redoano, 2002). But, again, this is not the crucial difference that most policy oriented observers seem to have in mind.

For example, in a very influential policy paper, Prud'homme (1994) severely warned against "the dangers of decentralization", the main danger exactly being the (presumed) stronger influence of local interest groups on local governments. Prud'homme's argument has nothing to do with preferences heterogeneity. It relies instead on a greater "disposition" by local governments to "accept" pressures from local interests, presumably due to the fact that supporting a local interest may generate additional benefits for the local politicians than supporting a foreign one.<sup>1</sup> This is exactly the same idea that continuously surfaces in the political debate, both at the national and at the international level. Is this idea correct? If the answer is yes, then there are additional reasons for, say, supporting centralization at the EU level in the above mentioned fields. If the answer is no, then there are additional reasons to maintain these functions at the level of member states.<sup>2</sup> And clearly, the EU is only an example, albeit an important one. Given the current process of decentralization in most countries, an answer to this question could be helpful in many other cases.<sup>3</sup>

To discuss this issue, we focus on a simple framework. In our model, there are two regions, one resident firm, and a large mass of consumers in each region owning the local firm. The two regional firms may serve both local markets, and in all cases they have an incentive to lobby the governments in charge either to gain access to local markets or to increase the production of a local public good which is complementary in consumption to the good they sell. As an interpretation, one could think of the former as an example of regulation policy, and of the latter as an example of infrastructure policy, both items over which there is some debate in Europe about which level of government should be in charge of these policies, whether the European Commission or the member countries. For simplicity, and also because these effects are well understood, we abstract entirely from "common pool" effects which may arise out of transfers from the central level to local ones (Persson, 1998), as well as from "fiscal competition" effects which may arise out of the mobility of the tax base (Wilson, 1999), or by "spillover effects" in taxation (Keen and Kotsogiannis, 2002). In our model, nobody moves, there is no need for intergovernmental transfers (as regions are identical), and each local government finances its supply of local public good out of resident taxation, so that there is no tax competition. However, there are spillover effects from regional public goods supply, as the latter affects firms' profits. Hence, when deciding about public good supply, while the central government internalizes as components of social welfare the profits that both firms make in both markets, under decentralization a local government is

<sup>&</sup>lt;sup>1</sup>We are not aware of any empirical work studying the relationship between decentralization and lobbying. There are, however, some empirical works discussing the relationship between corruption and decentralization, usually finding a negative correlation between the two. For recent examples, see Treisman (2000) and Fisman and Gatti (2002).

 $<sup>^{2}</sup>$ For instance, contrary to Prud'homme's claim, Shleifer and Vishny (1993), discussing corruption, argue that competition among local jurisdictions to attract businesses leads to a fall in corruption activity.

<sup>&</sup>lt;sup>3</sup>Spain, Italy, France, Belgium, to consider only some of the main European countries, have recently changed their Constitution in the direction of more decentralization. The UK, which has not a formal Constitution, has however launched the devolution process in Wales and Scotland.

only interested in the profits that are made everywhere by its own resident firm. This captures in the simplest way Prud'homme's idea that local interests may have a larger weight on local governments' welfare function.

In this setting, we ask what are the effects of lobbying on economic outcomes and social welfare in the two cases of decentralization and centralization. We consider two forms of lobbying. In the first one, firms lobby *in* the market; that is, both firms have already gained access to both markets and have an incentive to lobby politicians to increase local public good production. In this case, firms have *aligned interests* in increasing local public good production. In the second one, firms lobby *for* the market; that is, they lobby politicians to gain access to local markets, so that local firms have *conflicting interests*; each one wishes to be the only one to serve the market, as it could then raise monopoly profits.

We get very sharp results. When lobbying is in the market, lobbying behavior under centralization is always at least as bad for social welfare as under decentralization. In fact, under decentralization, when both firms lobby both local politicians, local public goods supply is as distorted as under centralization (and so is social welfare), but lobbies pay higher contributions and so are worse off. Moreover, under decentralization, depending on market structure, there are also equilibria in which each firm lobbies only one politician at the time, while this does not occur under centralization. In this case, contributions are lower and so are the distortions in social welfare. The intuition is that decentralization makes more difficult for lobbies to coordinate their actions and this increases the bribes they have to pay to force local governments to internalize spillover effects on profits; as a result, lobbying is less effective.

Results are reversed when lobbying is for the market. Under decentralization lobbying always leads the local politicians to give market access to the resident firm only, as the local firm can always outbid the foreign one, although a duopoly may be better for social welfare. This is so because only the local firm's profits enter into the social welfare function of the local government. Under centralization, this effect is absent, which makes the central politician more resilient to lobbying. Finally, we also show that in this case the most effective institutional structure against lobbying distortions may be an intermediate one between centralization and decentralization (i.e. one in which competencies are split between local and central governments).

The model we discuss in this paper is simple, but it offers far reaching insights. The basic message of the paper is that the answer to the question above depends on the function under consideration; if the interests of local lobbies are aligned, then decentralization is better than centralization because decentralization introduces competition across lobbies where there is none; vice-versa, if the interests of national lobbies are in conflict, then centralization is better than decentralization because, as Prud'homme rightly suggested, local governments are more easily captured by local interests. We will discuss some extensions and applications of these ideas for institutional design in the concluding section.

We model lobbying using the "reduced form" illustrated by Grossman and Helpman (2001) (see also Grossman and Helpman, 1994, and Dixit *et al.*, 1997). However, in the analysis of lobbying under decentralization, we go beyond the Common Agency approach of Bernheim and Whiston (1986a), by considering a case with multiple principals and multiple agents. Although we do not break new theoretical grounds on this issue, some of our findings can be taken as an extension of the recent literature (e.g. Prat and Rustichini, 2003, Segal, 1999).

The rest of the paper is organized as follows. In Section 2 we set up the model. In Section 3 we examine the policy makers' choices in the situation of no lobbying. In Section 4 we examine lobbying behavior when both firms compete and lobby in the market. In Section 5 we study lobbying for the market. Section 6 concludes by summarizing the results and suggesting avenues for further research. Most proofs and technical details are in two appendices.

# 2 The model

The economy is composed of two identical regions indexed by  $r \in \{a, b\}$ . There are four goods: two private consumption goods, x and z, a production factor, y, and a public investment good, g. The latter is purely local, meaning that there is a distinct provision in each region with no spillover effects across regions. In each region lives a continuum of immobile identical consumers with a mass of unity, and there is a firm producing good x, indexed by  $\rho \in \{\alpha, \beta\}$ , where  $\alpha$  and  $\beta$  are the firms located in regions a and b, respectively. In both regions consumers are endowed with a fixed quantity  $\bar{y} > 0$  of the production factor and have identical preferences represented by the quasi-linear utility function

$$u(x_r, z_r, g_r) = x_r - \frac{x_r^2}{2g_r} + z_r.$$
 (1)

We take good z to be the *numeraire* and its (local) market to be perfectly competitive. Technology is linear and units are normalized so that the production of one unit of z requires one unit of input y. These assumptions imply that at the market equilibrium profits in the production of good z are zero, that its supply is perfectly elastic, and that the market price of factor y is equal to one. Firms  $\alpha$  and  $\beta$  are entirely owned by consumers living in regions a and b, respectively, and their profits are entirely distributed to shareholders.<sup>4</sup> Hence, consumers' income is made up of two terms: the market value of the fixed endowment of good y, and the distributed firms' profits (net of contributions to the politicians, if any). Consumers' income in region r is subject to a proportional income tax at rate  $t_r, t_r \in [0, 1)$ . We let  $p_r$  be the price of good x in region r,  $\Pi_{\rho r}$  be the profits (gross of contributions) earned by firm  $\rho$  in region r, and  $s_{\rho r}$  be the contributions to politicians by firm  $\rho$  for public good  $g_r$ . Without loss of generality (given symmetry between regions), in what follows we only focus on region a. Let  $\pi_{\alpha} = \Pi_{\alpha a} - s_{\alpha a} + \Pi_{\alpha b} - s_{\alpha b}$  be the profits distributed by firm  $\alpha$ . Taking  $g_a$  and  $\pi_{\alpha}$  as given, each consumer in region a solves:

$$\max_{\substack{x_a, z_a \\ \text{s.t.}}} \quad x_a - \frac{x_a^2}{2g_a} + z_a,$$
  
s.t. 
$$p_a x_a + z_a \le (1 - t_a)(\bar{y} + \pi_\alpha),$$

from which we immediately obtain the inverse demand function for good  $x_a$  as

$$p_a(x_a, g_a) = 1 - \frac{x_a}{g_a}.$$
(2)

From (2) it is clear that for any given quantity  $x_a > 0$  an increase in  $g_a$  increases the marginal willingness to pay for good  $x_a$ .

#### **2.1** The markets for good x

In each region good x is traded in a local duopoly, with one of the firms located within the region and the other one outside it. Firms maximize profits and compete à la Cournot. Good y is the only input in production and technology is linear, so that marginal costs are constant. We allow however for a source of asymmetry between firms. When a firm supplies to its own regional market (at "home"), the production function is x = y/c (the marginal cost is c > 0), while when a firm supplies "abroad" the production function is  $x = y/(\delta c), \delta \ge 1$  (the marginal cost is  $\delta c$ ), so that the home firm has a cost advantage over its competitor. This assumption allows us to enquiry on the effects of market structure on lobbying behavior.<sup>5</sup>

<sup>&</sup>lt;sup>4</sup>Notice that given the quasi-linearity of the utility function, all income effects fall on the demand for good z, and therefore the equilibrium of the economy is independent of the distribution of profits across consumers and across regions. Also note that profit maximization by firms may be in conflict with shareholders' interests in their role as consumers, a standard problem in modelling the objectives of not competitive firms. We assume the existence of some un-modelled agency problem that justifies profit maximization by firms.

<sup>&</sup>lt;sup>5</sup>For instance, the parameter  $\delta$  (strictly speaking,  $\delta - 1$ ) can be interpreted as representing the transport costs needed to transfer one unit of good x across regions. Notice that our assumption of

Let  $x_{\rho r}$  be the quantity sold by firm  $\rho$  in region r; hence aggregate sales in regions a and b can be written as  $x_a = x_{\alpha a} + x_{\beta a}$  and  $x_b = x_{\alpha b} + x_{\beta b}$ . Using (2), firm  $\alpha$  then solves:

$$\max_{x_{\alpha a}, x_{\alpha b}} \quad \Pi_{\alpha a} + \Pi_{\alpha b} = \\
= \left(1 - \frac{x_{\alpha a} + x_{\beta a}}{g_a} - c\right) x_{\alpha a} + \left(1 - \frac{x_{\alpha b} + x_{\beta b}}{g_b} - \delta c\right) x_{\alpha b}.$$
(3)

Solving this problem and the symmetric one for firm  $\beta$ , we obtain the equilibrium quantities

$$x_{\alpha a}^{*} = hg_{a}, \quad x_{\beta b}^{*} = hg_{b}, \quad x_{\beta a}^{*} = fg_{a}, \quad x_{\alpha b}^{*} = fg_{b},$$
  
$$x_{a}^{*} = (h+f)g_{a}, \quad x_{b}^{*} = (h+f)g_{b},$$
  
(4)

and the equilibrium prices

$$p_a^* = p_b^* = p^*, \quad p^* = 1 - (h+f),$$

where

$$h = \frac{1 + \delta c - 2c}{3}, \qquad f = \frac{1 + c - 2\delta c}{3}.$$
 (5)

To ensure that the quantities (and the respective prices) supplied by each firm in each region are non-negative, we impose the following restrictions on parameters:

Assumption 1 0 < c < 1 and  $1 \le \delta \le \delta_{\max} = \frac{1+c}{2c}$ .

This framework allows for a wide range of market structures. When  $\delta = 1$ , h = f = (1 - c)/3, and there is a symmetric duopoly in each region, since the "home" firm has no cost advantage over its "foreign" rival. At the other extreme, when  $\delta = \delta_{\max}$ , h = (1 - c)/2 and f = 0. The cost advantage of the "home" firm is so high that the "foreign" firm does not enter the market, and thus there is a monopoly in each region. A continuum of intermediate cases is obtained for  $\delta \in (1, \delta_{\max})$ .

Finally notice that the equilibrium gross profits are linearly increasing in public good provision, so that firms' managers have an incentive to lobby the policy maker(s) for an expansion in the provision of the public goods:

$$\Pi_{\alpha}^{*} = \Pi_{\alpha a}^{*} + \Pi_{\alpha b}^{*} = h^{2}g_{a} + f^{2}g_{b}, \quad \Pi_{\beta}^{*} = \Pi_{\beta a}^{*} + \Pi_{\beta b}^{*} = f^{2}g_{a} + h^{2}g_{b}.$$
(6)

a given industrial structure (one resident firm in each region) could be justified by introducing fixed costs, which make unprofitable for a firm located in one region to open a new plant in the other region in spite of transportation costs. To keep the analysis simple, however, we do not model them explicitly.

#### 2.2 The public sector

We consider two institutional settings. One is a *centralized system*, in which a single policy maker chooses the supply of public goods in both regions. The other is a *decentralized* one, in which each region is characterized by an independent policy maker choosing the local level of the public good. In both cases we assume public goods production to be financed by a residence-based income-tax. Technology for public good production shows decreasing returns, with factor y used as the only input. The corresponding cost function is assumed to be of the form  $\phi g_r^2$ ,  $\phi > 0$ . To ease notation, and without loss of generality, we let  $\phi = 1/4$ .

Under a centralized system, a single decision maker chooses  $g_a$  and  $g_b$  and sets a uniform tax rate across regions,  $t_a = t_b = t$ .<sup>6</sup> The budget constraint is then:

$$\frac{g_a^2 + g_b^2}{4} = t(\pi_\alpha^* + \pi_\beta^* + 2\bar{y}),\tag{7}$$

where  $\pi_{\rho}^{*} = \Pi_{\rho}^{*} - s_{\rho a} - s_{\rho b}$ .

Under a decentralized system, each regional policy maker independently and simultaneously chooses public good provision in her own region, and public expenditure is financed through the local income tax. The regional budget constraints are then:

$$\frac{g_a^2}{4} = t_a(\pi_\alpha^* + \bar{y}), \quad \frac{g_b^2}{4} = t_b(\pi_\beta^* + \bar{y}). \tag{8}$$

Notice that by Walras' law, satisfaction of the relevant government budget constraints implies that the markets for good z and factor y also clear, under both centralization and decentralization.<sup>7</sup>

## 2.3 Social welfare

To compare the alternative institutional arrangements, we need a normative criteria. We then define social welfare as the sum of consumers' surplus, distributed profits, and the contributions raised by the government.<sup>8</sup> Substituting the equilibrium values for

<sup>&</sup>lt;sup>6</sup>Under centralization, uniformity of regional income tax rates is a natural assumption. Furthermore, in many countries, discrimination of the income tax on territorial bases is forbidden by the Constitution.

<sup>&</sup>lt;sup>7</sup>The supply of good z is perfectly elastic and thus its equilibrium quantity is determined by national demand,  $z^d$ , from consumers. As for factor y, national supply from consumers is inelastic,  $y^s = 2\bar{y}$ . The demand for y comes from three sources: the public sector  $(y_{PS}^d)$ , the firms producing good z  $(y_Z^d)$ , and the firms  $\alpha$  and  $\beta$   $(y_{\alpha+\beta}^d)$ . By Walras' law, given that the centralized public sector's budget constraint balances, it follows that  $y_Z^d + y_{PS}^d + y_{\alpha+\beta}^d = y^s$ , where  $y_{PS}^d = (g_a^2 + g_b^2)/4$ ,  $y_Z^d = z^d = 2\bar{y} + \pi_{\alpha}^* + \pi_{\beta}^* - p^*(x_a^* + x_b^*)$ ,  $y_{\alpha+\beta}^d = c(x_{\alpha a}^* + \delta x_{\alpha b}^* + x_{\beta b}^* + \delta x_{\beta a}^*)$ . The same holds under decentralization.

<sup>&</sup>lt;sup>8</sup>This definition makes contributions from the home firm to the local politician a pure transfer, with no effects on social welfare. As an alternative, we could have excluded contributions from social welfare,

 $x_a^*, z_a^* = -p^* x_a^* + (1 - t_a)(\bar{y} + \pi_\alpha^*)$ , and  $\pi_\alpha^*$  into the utility function of consumers (1), social welfare in region *a* is then

$$\mathcal{W}_a = x_a^* - \frac{(x_a^*)^2}{2g_a} - p^* x_a^* + (1 - t_a)(\bar{y} + \Pi_{\alpha a}^* - s_{\alpha a} + \Pi_{\alpha b}^* - s_{\alpha b}) + s_{\alpha a} + s_{\beta a},$$

which using (4), (6), and (8), can be rewritten as

$$\mathcal{W}_a(g_a, g_b) = W_a(g_a, g_b) - s_{\alpha b} + s_{\beta a},\tag{9}$$

where

$$W_a(g_a, g_b) = \frac{(h+f)^2 g_a + 2(h^2 g_a + f^2 g_b)}{2} - \frac{g_a^2}{4} + \bar{y}.$$
 (10)

National social welfare,  $\mathcal{W} = \mathcal{W}_a + \mathcal{W}_b$ , is then

$$\mathcal{W}(g_a, g_b) = \frac{(h+f)^2 + 2(h^2 + f^2)}{2}(g_a + g_b) - \frac{g_a^2 + g_b^2}{4} + 2\bar{y}.$$
(11)

Notice that the net effect of lobbyists' contributions on *national* social welfare is nil, since they are a pure transfer from lobbyists to politicians. This is no longer true under decentralization. In this case, a contribution of firm  $\alpha$  to the policy maker of region b counts as a welfare loss in region a, whereas a contribution of firm  $\beta$  to the policy maker of region a counts as a welfare gain in region a.

# 3 Public good provision without lobbying

Let us begin our analysis by examining policy choices in the case of no lobbying. Under centralization, the policy maker would choose public goods supply by maximizing (11), giving for both  $g_a$  and  $g_b$ :<sup>9</sup>

$$\hat{g}^C = (h+f)^2 + 2(h^2 + f^2).$$
 (12)

Under decentralization, on the other hand, the policy maker of region a would maximize (10) with respect to  $g_a$ , taking  $g_b$  as given (and an analogous problem is solved by the policy maker in region b), obtaining the symmetric solution

$$\hat{g}^D = (h+f)^2 + 2h^2.$$
 (13)

and then let contributions enter the choice function of the politician (see Eq. 17 below) as a separate component only. The main qualitative results concerning lobbying behavior in the two institutional settings would remain unchanged under this alternative definition, as it would still be true that under decentralization and lobbying local politicians assign different weights to the contributions by the home and the foreign firm, a fact that leads our results. Details are available by the authors on request.

<sup>&</sup>lt;sup>9</sup>To ease notation, throughout the paper, we suppress pedices for regions (a, b) and for firms  $(\alpha, \beta)$  whenever by symmetry the corresponding magnitudes are identical.

By using (6), (12) and (13), equilibrium profits of each firm under centralization and decentralization are:

$$\hat{\pi}^C = (h^2 + f^2)\hat{g}^C, \tag{14}$$

$$\hat{\pi}^D = (h^2 + f^2)\hat{g}^D.$$
(15)

It follows:

**Proposition 1** Suppose there is no lobbying. Then if  $\delta \in [1, \delta_{\max})$  public good supply, national social welfare and firms' profits are higher under centralization than under decentralization. In the limiting case  $\delta = \delta_{\max}$ , the two regimes are equivalent.

**Proof.** The part on public good supply and firms' profits follows from  $f^2 > 0$  if  $\delta \in [1, \delta_{\max})$  and  $f^2 = 0$  if  $\delta = \delta_{\max}$ , and by comparison of (12)–(13) and of (14)–(15), respectively. As for aggregate social welfare, since  $g_a = g_b = \hat{g}^C$  is a global maximum of (11), the latter is not maximized for  $g_a = g_b = \hat{g}^D < \hat{g}^C$ .

The intuition is simple. Without lobbying, when the regional policy maker decides about local public good supply, she does not internalize as social welfare gains the profits made by the non-resident firm. Hence, when both firms sell in both regions, local public good supply is lower under decentralization and so are profits and national welfare. On the contrary, a centralized policy maker internalizes both firms' profit gains, and hence she has a greater incentive to expand public good supply. These incentives are the same when the resident firm is a monopoly within its own region, and hence  $\hat{g}^C = \hat{g}^D$ .<sup>10</sup>

# 4 Lobbying in the market

We now consider the effect of introducing lobbying into the model. We examine two different cases, lobbying *in* the market and lobbying *for* the market. In the first case, firms are already present in the market and have an incentive to lobby politicians to increase public good supply as this increases their profits. In the second case, firms compete to acquire the right to enter the market. In both cases, we derive equilibrium contributions and public goods supply under centralization and decentralization, and compare the results on normative grounds.

<sup>&</sup>lt;sup>10</sup>One might wonder whether the fact that, without lobbying, centralization performs better than decentralization impairs our comparison in the context of lobbying. This does not occur since in our model the benchmark case is the one in which there is lobbying activity *and* policy makers are fully benevolent, a situation in which centralization and decentralization are indeed equivalent regimes, as it will be discussed below.

In this section we analyze the case of lobbying in the market, studying lobbying behavior in the *common agency* framework developed by Bernheim and Whinston (1986b). Notice, however, that under decentralization, as there are two principals (firms  $\alpha$  and  $\beta$ ) lobbying two agents (policy makers *a* and *b*), our model falls into the more general category of the so called *games played through agents*, recently investigated by Prat and Rustichini (2003). We begin with the centralized system.

## 4.1 Centralization

We assume that the policy maker maximizes a weighted average of social welfare and lobbyists' contributions. This is clearly a "reduced form" of a more complex (and unmodelled) political behavior. Politicians care for social welfare (presumably, because they want to be re-elected), but they also care for lobbyists' contributions, either because the latter increase their chances of being re-elected, or simply because these are bribes which increase the policy maker's private consumption. Each lobby maximizes profits net of the contributions to the policy maker. As for the timing, firms move first, by independently and simultaneously offering the policy maker a contribution schedule defining a monetary contribution as a function of public good provision (lobbies can commit to this contribution schedule). Upon acceptance of the lobbies contributions, the policy maker chooses public goods supply.

Following Dixit *et al.* (1997), we focus on *truthful* subgame perfect Nash equilibria, in which each lobby offers the policy maker a non-negative *compensating contribution schedule*, shaped along its iso-profit curve. Firms  $\alpha$  and  $\beta$  compensating contribution schedules are then defined, respectively, as

$$S_{\alpha}(g_a, g_b, \pi_{\alpha}) = \max\left\{h^2 g_a + f^2 g_b - \pi_{\alpha}, 0\right\},$$
(16a)

$$S_{\beta}(g_a, g_b, \pi_{\beta}) = \max\left\{f^2 g_a + h^2 g_b - \pi_{\beta}, 0\right\}.$$
(16b)

Firm  $\rho$  defines its strategy by choosing its net profits,  $\pi_{\rho}$ , which in turn determine the "position" of its contribution schedule in the  $(s_{\rho}, g_a, g_b)$  hyperplane. Obviously, it must hold in equilibrium that  $\pi_{\rho} \geq \hat{\pi}^C$ , since otherwise the firm would prefer not to lobby.

Using (11) and (16a)–(16b), the policy maker's objective function is

$$V^{C}(g_{a}, g_{b}, \pi_{\alpha}, \pi_{\beta}) = \mu \mathcal{W}(g_{a}, g_{b}) + (1 - \mu) \left[ S_{\alpha}(g_{a}, g_{b}, \pi_{\alpha}) + S_{\beta}(g_{a}, g_{b}, \pi_{\beta}) \right].$$
(17)

The parameter  $\mu$ ,  $0 < \mu \leq 1$ , captures the degree of "benevolence" of the policy maker. By assuming  $\mu \neq 0$ , we rule out the unrealistic case in which the politician cares about contributions only.

We illustrate here the outcome of the lobbying game under centralization, leaving to the Appendix A.1 the analytical details. The game is solved by backward induction. In



Figure 1: Nash equilibria under centralization

the second stage, given  $S_{\alpha}(.)$  and  $S_{\beta}(.)$ , the policy maker chooses  $g_a$  and  $g_b$  to maximize (17). The optimal public good supply, both for  $g_a$  and  $g_b$ , is<sup>11</sup>

$$\tilde{g}^C = \hat{g}^C + 2m(h^2 + f^2), \tag{18}$$

where

$$m = \frac{1-\mu}{\mu}.$$
(19)

Unsurprisingly, lobbying induces an upward distortion in public good supply, and hence a social welfare loss, unless the policy maker is fully benevolent ( $\mu = 1$ ). Notice also that  $\tilde{g}^{C}$  is invariant to the choice of  $\pi_{\alpha}$  and  $\pi_{\beta}$  by firms in stage one. This is so because contributions enter linearly into the policy maker preferences (17), and therefore  $\pi_{\alpha}$ and  $\pi_{\beta}$  have no income effects on the choice of  $g_{a}$  and  $g_{b}$ .

At stage one, each firm  $\rho$ , given the strategy of the other firm, maximizes its net profits  $\pi_{\rho}$  subject to the politician's participation constraint. As we show in Appendix A.1, the lobbying game under centralization admits a multiplicity of Nash equilibria, which are illustrated in Figure 1. The solid curve depicts firm  $\beta$  best response function,  $\pi^*_{\beta}(\pi_{\alpha})$ , whereas the dotted curve represents firm  $\alpha$  best response function,  $\pi^*_{\alpha}(\pi_{\beta})$ . The set of Nash equilibria lies along the segment E'-E''. Notice, however, that since the slope of the best response functions is equal to -1 along E'-E'', all Nash equilibria are equivalent in terms of aggregate profits (and contributions). The multiplicity of Nash equilibria is due to the linearity of firms profits and politician's preferences in contributions. Hence, given the aggregate rent that lobbies can extract from the policy

<sup>&</sup>lt;sup>11</sup>Throughout the paper a "hat" denotes the solutions obtained without lobbying, whereas a "tilde" denotes the corresponding solutions under lobbying in the market.

maker, there exist several ways to allocate this amount between the two players, as an increase of the contribution by one firm, matched with a corresponding reduction of the contribution by the other firm, bears no income effects on all players.

Since aggregate profits are constant and firms are identical, in the following we restrict our attention to the symmetric equilibrium  $E^C$  in Figure 1, in which  $\tilde{\pi}^C_{\alpha} = \tilde{\pi}^C_{\beta} = \tilde{\pi}^C$ . As shown in Appendix A.1, in the symmetric equilibrium, net profits and contributions are

$$\tilde{\pi}^C = \hat{\pi}^C + m(h^4 + f^4 + 2h^2 f^2), \tag{20}$$

$$\tilde{s}^C = m(h^4 + f^4 + 2h^2 f^2). \tag{21}$$

Eq. (20) shows that profits under lobbying are equal to the profits without it,  $\hat{\pi}^C$ , plus a profit gain from lobbying. As expected, if the policy maker does not care for lobbyists' contributions (i.e. m = 0),  $\tilde{\pi}^C = \hat{\pi}^C$  and  $\tilde{s}^C = 0$ .

The lobbying game we have considered above, in which both firms lobby for both public goods, is not the only conceivable one. In principle, each firm has four different options — lobby for both public goods, lobby for one public good only, and no lobby. However, we do not need to examine all the corresponding games, since each firm's profits are larger if it lobbies for both public goods, no matter what the other firm does. This follows directly from the definition of truthful strategy and the associated compensating contribution function. From Proposition 2 in Dixit *et al.* (1997), a truthful strategy is weakly dominant, and in our setting truthful strategies always involve non-negative contributions by both firms on both public goods. As we show in the next section, the same does not hold true under decentralization.

## 4.2 Decentralization

The structure of the lobbying game under decentralization is more complex. The game is set in three stages. At stage one, each firm chooses among four possible strategies: lobby *both* regions (B), lobby only "at home" — *inside* its region (I), lobby only "abroad" — *outside* its region (O), and, finally, *no* lobby (N). This strategy set gives rise to a  $4 \times 4$  normal form symmetric game that we denote as the *where-tolobby* game. Given the strategies chosen at stage one, at stage two firms play the corresponding lobbying game. In this game, as in the previous section, lobbies move first by committing to a contribution schedule for the policy makers as a function of public good choices, and then policy makers choose public goods supply. For instance, if at stage one firm  $\alpha$  chooses I and firm  $\beta$  plays B, at stage two,  $\alpha$  and  $\beta$  simultaneously and independently present the policy maker in region a contribution schedule as a function of  $g_a$ , while at the same time  $\beta$  offers the policy maker of region *b* a contribution schedule as a function of  $g_b$ . Finally, at stage three, given the contribution schedules, policy makers simultaneously and independently choose public good supply in their own region. For each one of the 16 strategy pairs of the where-to-lobby game, the payoffs are thus given by firms net profits at the truthful equilibrium of the corresponding lobbying-game. By symmetry, however, to solve the where-to-lobby game it is sufficient to consider 9 different games (in addition to the no-lobby case already examined in Section 3). In what follows, we solve the where-to-lobby game by proceeding backwards, solving first for equilibrium contributions and profits in each of the 9 games, and then finding where firms prefer to lobby by deriving the Nash equilibria of the where-tolobby game. We focus here only on the results of the analysis; details of proofs are contained in Appendix A.2.

#### Characterization of the various lobbying games

Let  $S_{\rho r}(g_r, \pi_{\rho r})$  be the compensating contribution schedule that firm  $\rho$  offers the policy maker of region r, where  $S_{\alpha a} = \max \{h^2 g_a - \pi_{\alpha a}, 0\}, S_{\beta a} = \max \{f^2 g_a - \pi_{\beta a}, 0\}, S_{\alpha b} = \max \{f^2 g_b - \pi_{\alpha b}, 0\}$  and  $S_{\beta b} = \max \{h^2 g_b - \pi_{\beta b}, 0\}.$ 

Suppose that at stage one both firms have chosen strategy (BB), that is they both lobby in both regions (BB). Given the contribution schedules  $S_{\alpha a}(.)$ ,  $S_{\beta a}(.)$ ,  $S_{\alpha b}(.)$ and  $S_{\beta b}(.)$ , at stage three policy makers a and b solve, respectively,<sup>12</sup>

$$\tilde{g}_a^{BB} = \underset{g_a}{\operatorname{argmax}} V_a^{BB}(g_a, g_b, \pi_{\alpha a}, \pi_{\beta a}, \pi_{\alpha b}),$$
(22a)

$$\tilde{g}_b^{BB} = \underset{g_b}{\operatorname{argmax}} V_b^{BB}(g_a, g_b, \pi_{\beta b}, \pi_{\alpha b}, \pi_{\beta a}),$$
(22b)

where

$$\begin{split} V_{a}^{BB}(.) &\equiv \mu (W_{a} - S_{\alpha b} + S_{\beta a}) + (1 - \mu) (S_{\alpha a} + S_{\beta a}), \\ V_{b}^{BB}(.) &\equiv \mu (W_{b} - S_{\beta a} + S_{\alpha b}) + (1 - \mu) (S_{\beta b} + S_{\alpha b}), \\ \tilde{V}_{a}^{BB} &\equiv V_{a}^{BB}(\tilde{g}_{a}^{BB}, \tilde{g}_{b}^{BB}, \pi_{\alpha a}, \pi_{\beta a}, \pi_{\alpha b}), \ \tilde{V}_{b}^{BB} &\equiv V_{b}^{BB}(\tilde{g}_{a}^{BB}, \tilde{g}_{b}^{BB}, \pi_{\beta b}, \pi_{\alpha b}, \pi_{\beta a}). \end{split}$$

As already noted, under decentralization different lobbies' contributions do not have the same weight into the local politicians' preferences. One unit of contribution a firm makes abroad counts as  $-\mu$  in the home region but as 1 in the recipient region, while one unit of contribution a firm makes at home counts as  $1 - \mu$  in the home region and nothing abroad.

<sup>&</sup>lt;sup>12</sup>We assume that the degree of benevolence of regional policy makers is the same as that of the central policy maker.



Figure 2: Nash equilibria when both firms lobby in region a under decentralization

Solving the problem, the optimal public good supply in each region is (see Appendix A.2)<sup>13</sup>

$$\tilde{g}^{BB} = \hat{g}^D + 2f^2 + 2m(h^2 + f^2).$$
(24)

Notice that in a decentralized system the lobbies influence public policy even when the social planner is fully benevolent (i.e. m = 0, since  $\mu = 1$ ), as can be seen by the second term in (24). Moreover, in this case public good supply is the same as under centralization ( $\tilde{g}^{BB} = \tilde{g}^{C}$ ), which in turn equals the supply under centralization without lobbying ( $\hat{g}^{C}$ ). The reason is simple. Under decentralization, the lobbies, by offering truthful contribution schedules to benevolent policy makers, induce them to internalize the spillover effects on profits of public good supply, thus providing the proper incentives to set the same (optimal) level of public good provision that occurs under centralization.<sup>14</sup> This also implies that the case of lobbying a fully benevolent planner can be taken as the benchmark in our context, allowing us to meaningfully compare the two institutional settings when governments are not fully benevolent.

At stage two, each firm, given the strategies of the other firm, maximizes its net profits subject to the politicians' participation constraints. As we show in Appendix A.2, the lobbying game *BB* admits a multiplicity of Nash equilibria in each region.<sup>15</sup> Segment E'-E'' in Figure 2 represents the set of Nash equilibria occurring in region *a*; the solid curve is firm  $\beta$  best response function,  $\pi^*_{\beta a}(\pi_{\alpha a})$ ; the dotted curve is firm

<sup>&</sup>lt;sup>13</sup>As under centralization,  $\tilde{g}^{BB}$  is independent of  $\pi_{\alpha a}$ ,  $\pi_{\alpha b}$ ,  $\pi_{\beta b}$  and  $\pi_{\beta a}$ , as a consequence of the linearity of politicians' preferences in contributions.

<sup>&</sup>lt;sup>14</sup>Truthful contribution schedules work as a standard *Pigouvian* subsidy scheme.

<sup>&</sup>lt;sup>15</sup>As under centralization, multiplicity of equilibria is due to linearity of firms net profits and politicians' preferences in contributions.

 $\alpha$  best response function,  $\pi_{\alpha a}^*(\pi_{\beta a})$ . An identical (symmetric) set of Nash equilibria is obtained in region *b* for net profits  $\pi_{\beta b}$  and  $\pi_{\alpha b}$ . Notice also that firms play a "separate" game in each region: the best response function of firm  $\rho$  in region *r* does not depend on its own and the other firm best response in the other region. This result comes again from the linearity of firms profits and politicians' preferences in contributions. Formally, by symmetry between regions, the set of Nash equilibria in terms of firms net profits can be characterized by the parameter  $\sigma^{BB} \in [0, 1]$ , such that

$$\tilde{\pi}_{\beta a} = \tilde{\pi}_{\alpha b} = f^2 \hat{g}^D + (1+m) f^4 + 2mh^2 f^2 \sigma^{BB},$$
(25)

$$\tilde{\pi}_{\alpha a} = \tilde{\pi}_{\beta b} = h^2 \hat{g}^D + mh^4 + 2(1+m)h^2 f^2 (1-\sigma^{BB}),$$
(26)

from which equilibrium contributions are obtained as

$$\tilde{s}^{BB}_{\beta a} = \tilde{s}^{BB}_{\alpha b} = (1+m)f^4 + 2mh^2 f^2 (1-\sigma^{BB}),$$
  
$$\tilde{s}^{BB}_{\alpha a} = \tilde{s}^{BB}_{\beta b} = mh^4 + 2(1+m)h^2 f^2 \sigma^{BB}.$$

Clearly, lobbies have a distributive conflict in each region. They both gain from an increase in public good production in each region, but each one would prefer the other firm to pay the contributions to the local politician. This conflict is formally captured by the parameter  $\sigma^{BB}$  characterizing the set of Nash equilibria. At one end, when  $\sigma^{BB} = 1$ , the equilibrium is E', the one preferred by the foreign lobby; at the other end, when  $\sigma^{BB} = 0$ , the equilibrium is E'', the one preferred by the home lobby.<sup>16</sup> This conflict was present even under centralization (see the previous section), but there is now an important difference. Under decentralization, the slope of the best response functions is less than one, so that aggregate profits in each region are decreasing in  $\sigma^{BB}$ . Figure 2 illustrates the point, by showing that aggregate profits in region a,  $\pi_{\alpha a} + \pi_{\beta a}$ , increase as one moves from E' to E''. This is due to the different value regional policy makers attach to contributions from the foreign and the home lobby. Consider for instance the equilibrium E'. Starting from this point, it is possible to select another equilibrium by increasing the contributions of the foreign firm ( $\beta$ ) by m euros while reducing those of the home firm ( $\alpha$ ) by 1 + m euros; hence, in the new equilibrium aggregate profits are higher than in E'. The policy maker in region a, although "losing" one euro of contributions, is indifferent, for she values m euros from the foreign firm as 1 + m euros from the home firm.

This fact brings about another important property of the game *BB*. Adding (25) and (26), the total (home plus abroad) net profits of each firm, defined as  $\tilde{\pi}^{BB} \equiv$ 

<sup>&</sup>lt;sup>16</sup>Since firms are identical, we have assumed that the parameter  $\sigma^{BB}$  is the same in both regions. By assuming a different  $\sigma_r^{BB}$  in each region, the support of the set of Nash equilibria would expand from  $\mathbb{R}$  to  $\mathbb{R}^2$ .

 $\tilde{\pi}_{\alpha a} + \tilde{\pi}_{\alpha b} \equiv \tilde{\pi}_{\beta b} + \tilde{\pi}_{\beta a}$ , are equal to

$$\tilde{\pi}^{BB} = \hat{\pi}^D + mh^4 + (1+m)f^4 + 2mh^2f^2 + 2h^2f^2(1-\sigma^{BB}).$$
(27)

Hence, total profits of both firms increase as  $\sigma^{BB}$  becomes lower. Hence, if firms succeed in coordinating their lobbying activity so as to play the equilibrium  $\sigma^{BB} = 0$  in each region, they attain an outcome that maximizes aggregate profits. At this equilibrium, firm  $\alpha$  pays most part of the contributions in region b, and  $\beta$  does the same in region a. This is a crucial difference with the BB game under centralization examined in Section 4.1. In the latter, since the policy maker perceives the two lobbies as identical, the resulting Nash equilibria are all equivalent in terms of aggregate firms profits and contributions. On the contrary, under decentralization, there emerges a Pareto ranking of Nash equilibria in terms of firms' net profits. As  $\sigma^{BB}$  becomes lower, both firms profits increase, contributions decrease (thus making the policy makers worse off), whereas public good supply is fixed at the level shown in (24).

Having completed the description of the game BB, the equilibrium profits (27) are used to fill the corresponding cell of the where-to-lobby game in Table 1.<sup>17</sup> In the table, payoffs are shown as profit gains from lobbying; for instance,  $\Delta \pi_{\rho}^{BB} = \tilde{\pi}^{BB} - \hat{\pi}^{D}$  in the cell *BB*. In general,  $\Delta \pi_{\rho}^{ij} = \tilde{\pi}_{\rho}^{ij} - \hat{\pi}^{D}$  denotes the profit gains from lobbying by firm  $\rho$ when firm  $\alpha$  plays strategy *i* while firm  $\beta$  plays *j*, with  $i, j \in \{B, I, O, N\}$ .

Consider next the games IB, OB and OI (by symmetry, games BI, BO and IO are identical). The common feature of these games is that both firms lobby in one region, whereas there is at most one firm lobbying in the other region. Joint lobbying in one region brings about multiple Nash equilibria of the same type as those occurring in each region in the game BB (see Appendix A.2 ). Hence, by replacing  $\sigma^{BB}$  with  $\sigma^{ij}$ ,  $ij \in \{IB, OB, OI\}$ , Eqs. (25)–(26) define the set of Nash equilibria in the region lobbed by both firms. In the region in which there is only one firm lobbying (or no lobbying at all), the equilibrium is in all cases unique, and independent of the equilibrium occurring in the other region. This asymmetry — two lobbies in one region and at most one in the other — accounts for an important difference between games IB, OB and OI on one side, and game BB on the other. While in the BB game lobbies have an incentive to coordinate on the equilibrium  $\sigma^{BB} = 0$  in both regions, in the other games they only have conflicting interests in the region where they both lobby. In this region, in fact, the home (foreign) firm profits are decreasing (increasing) in  $\sigma^{ij}$ . For instance, in the cell OI of the where-to-lobby game, where firm  $\alpha$  lobbies only abroad and firm  $\beta$ 

<sup>&</sup>lt;sup>17</sup>The payoff of the row player, firm  $\alpha$ , is at the top, whereas that of the column player, firm  $\beta$ , is at the bottom.

		В	Ι	0	N		
		$\Delta \pi^{BB}_{\alpha} = mh^4 + (1+m)f^4 + 2mh^2 f^2 +$	$\Delta \pi^{BI}_{\alpha} = mh^4 + (1+m)f^4 +$	$\Delta \pi^{BO}_{\alpha} = mh^4 + (1+m)f^4 +$			
	$+2h^2f^2(1-\sigma^{BB}),$		$+2mh^2f^2\sigma^{BI},$	$+(2+2m)h^2f^2(1-\sigma^{BO}),$	$\Delta \pi^{BN}_{\alpha} = mh^4 + (1+m)f^4,$		
	В						
		$mh^4 + (1+m)f^4 + 2mh^2f^2 +$	$mh^4 + 2mh^2f^2 +$	$(2+2m)h^2f^2 + (1+m)f^4 +$	$(2+4m)h^2f^2 = \Delta\pi^{B\!N}_\beta$		
		$+2h^2f^2(1-\sigma^{BB})=\Delta\pi^{BB}_\beta$	$+(2+2m)h^2f^2(1-\sigma^{BI}) = \Delta\pi^{BI}_{\beta}$	$+2mh^2f^2\sigma^{BO}=\Delta\pi^{BO}_\beta$			
		$\Delta \pi^{I\!B}_{\alpha} = mh^4 + 2mh^2 f^2 +,$					
		$+(2+2m)h^2f^2(1-\sigma^{IB}),$	$\Delta \pi^{II}_{\alpha} = mh^4 + 2mh^2 f^2,$	$\Delta \pi^{IO}_{\alpha} = mh^4 + (2+2m)h^2 f^2 (1-\sigma^{IO}),$	$\Delta \pi^{I\!N}_{\alpha} = m h^4,$		
f	Ι						
i		$mh^4 + (1+m)f^4 +$	$mh^4 + 2mh^2 f^2 = \Delta \pi^{II}_\beta$	$(1+m)f^4 + 2mh^2 f^2 \sigma^{IO} = \Delta \pi^{IO}_\beta$	$2mh^2f^2=\Delta\pi_\beta^{I\!N}$		
r		$+2mh^2f^2\sigma^{IB}=\Delta\pi^{IB}_\beta$					
m		$\Delta \pi^{OB}_{\alpha} = (2+2m)h^2 f^2 + (1+m)f^4 +$					
$\alpha$		$+2mh^2f^2\sigma^{OB},$	$\Delta \pi^{OI}_{\alpha} = (1+m)f^4 + 2mh^2 f^2 \sigma^{OI},$	$\Delta \pi_{\alpha}^{OO} = (2+2m)h^2 f^2 + (1+m)f^4,$	$\Delta \pi^{ON}_{\alpha} = (1+m)f^4,$		
	0						
		$mh^4 + (1+m)f^4 +$	$mh^4 + (2+2m)h^2 f^2 (1-\sigma^{OI}) = \Delta \pi_{\beta}^{OI}$	$(2+2m)h^2f^2 + (1+m)f^4 = \Delta \pi_{\beta}^{OO}$	$(2+2m)h^2f^2 = \Delta\pi^{ON}_\beta$		
		$+(2+2m)h^2f^2(1-\sigma^{OB}) = \Delta\pi^{OB}_{\beta}$					
		$\Delta \pi^{NB}_{\alpha} = (2+4m)h^2 f^2,$	$\Delta \pi_{\alpha}^{NI} = 2mh^2 f^2,$	$\Delta \pi_{\alpha}^{NO} = (2+2m)h^2 f^2,$	0,		
	Ν						
		$mh^4 + (1+m)f^4 = \Delta \pi_\beta^{NB}$	$mh^4 = \Delta \pi^{NI}_eta$	$(1+m)f^4 = \Delta \pi_\beta^{NO}$	0		

	Γ	a	b]	$\mathbf{e}$	1:	Τ	The	W	here-to	-lo	$\mathbf{b}$	by	game	und	er d	lecentra	lizat	tior	l
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lobbies only at home, the payoff of the foreign firm,  $\Delta \pi_{\alpha}^{OI}$ , is increasing in  $\sigma^{OI}$  while that of the home firm,  $\Delta \pi_{\beta}^{OI}$ , is decreasing in  $\sigma^{OI}$ .

Games NB, NI and NO (by symmetry, games BN, IN and ON are identical) admit a unique Nash equilibrium, since there is at most one lobby in each region (see Appendix A.2). Here we focus on the remaining games II and OO.

Consider first the case in which both firms lobby their home region only (II). Policy makers solve

$$\tilde{g}_a^{II} = \underset{g_a}{\operatorname{argmax}} V_a^{II}(g_a, g_b, \pi_{\alpha a}), \tag{28a}$$

$$\tilde{g}_b^{II} = \underset{g_b}{\operatorname{argmax}} V_b^{II}(g_a, g_b, \pi_{\beta b}), \tag{28b}$$

where

$$V_{a}^{II}(.) \equiv \mu W_{a} + (1-\mu)S_{\alpha a}, \ \tilde{V}_{a}^{II} \equiv V_{a}^{II}(\tilde{g}_{a}^{II}, \tilde{g}_{b}^{II}, \pi_{\alpha a}),$$
  
$$V_{b}^{II}(.) \equiv \mu W_{b} + (1-\mu)S_{\beta b}, \ \tilde{V}_{b}^{II} \equiv V_{b}^{II}(\tilde{g}_{a}^{II}, \tilde{g}_{b}^{II}, \pi_{\beta b}).$$

As shown in Appendix A.2, public good supply, total net profits and contributions are, respectively,

$$\tilde{g}^{II} = \hat{g}^D + 2mh^2,\tag{30}$$

$$\tilde{\pi}^{II} = \hat{\pi}^D + mh^4 + 2mh^2 f^2, \quad \tilde{s}^{II} = mh^4.$$
(31)

When both firms lobby only abroad (OO), policy makers solve

$$\tilde{g}_a^{OO} = \underset{g_a}{\operatorname{argmax}} V_a^{OO}(g_a, g_b, \pi_{\beta a}, \pi_{\alpha b}),$$
(32a)

$$\tilde{g}_b^{OO} = \underset{g_b}{\operatorname{argmax}} V_b^{OO}(g_a, g_b, \pi_{\alpha b}, \pi_{\beta a}), \tag{32b}$$

where

$$V_{a}^{OO}(.) \equiv \mu(W_{a} - S_{\alpha b} + S_{\beta a}) + (1 - \mu)S_{\beta a}, \ \tilde{V}_{a}^{OO} \equiv V_{a}^{BB}(\tilde{g}_{a}^{OO}, \tilde{g}_{b}^{OO}, \pi_{\beta a}, \pi_{\alpha b}),$$
$$V_{b}^{OO}(.) \equiv \mu(W_{b} - S_{\beta a} + S_{\alpha b}) + (1 - \mu)S_{\alpha b}, \ \tilde{V}_{b}^{OO} \equiv V_{b}^{OO}(\tilde{g}_{a}^{OO}, \tilde{g}_{b}^{OO}, \pi_{\alpha b}, \pi_{\beta a}).$$

The solutions are

$$\tilde{g}^{OO} = \hat{g}^D + 2f^2 + 2mf^2, \tag{34}$$

$$\tilde{\pi}^{OO} = \hat{\pi}^D + 2(1+m)h^2f^2 + (1+m)f^4, \quad \tilde{s}^{OO} = (1+m)f^4.$$
 (35)

Notice that in the case of a benevolent policy maker (m = 0), the outcome of the lobbying game OO is the same as that of the game BB (the latter collapses into the former since equilibrium contributions paid at home are zero for m = 0), which is, as remarked above, the same (efficient) outcome emerging under centralization.

#### The solutions of the where-to-lobby game

Having derived the equilibrium profits and contributions for all possible lobbying games (the complete set of results is given in Tables A.1 and A.2, Appendix A.2), we have completed the construction of the where-to-lobby game in Table 1. We can now examine firms' choices at stage one. However, a general characterization of the where-to-lobby game outcomes would be a daunting task. Indeed, 7 cells of the game (*BB*, *IB*, *OB*, *OI*, *BI*, *BO* and *IO*) have a multiplicity of equilibrium payoffs, depending on the associated profit-sharing parameter  $\sigma^{ij}$ .

To reduce the set of possible outcomes, let us begin by assuming a uniform profit sharing parameter, i.e.

# Assumption 2 $\sigma^{ij} = \sigma \in [0,1], ij \in \{BB, IB, OB, OI, BI, BO, IO\}.$

One possible justification for this restriction is grounded in the "separability" properties of the model, already noted above. In fact, whenever the two firms lobby in one region, the structure of the game in that region is invariant to what goes on in the other region. Hence, we might expect that contributions by the home and the foreign firm end up allocated in the same way in all relevant cells, which implies a uniform profit sharing parameter. Before presenting the results, we also introduce an additional (and innocuous) assumption to describe lobbies' behavior when they are indifferent:

Assumption 3 Whenever not lobbying (N) and lobbying only at home (I) earns the same profit, a firm is assumed not to lobby. Whenever lobbying only at home (I) and lobbying only abroad (O) earns the same profit, a firm lobbies only at home. Whenever lobbying only abroad (O) and lobbying in both regions (B) earns the same profits, a firm is assumed to lobby only abroad.

The next proposition shows how the (unique) Nash equilibrium of the where-tolobby game is related to the values taken by the parameters  $\sigma$ ,  $\mu$  and  $\delta$ .

## Proposition 2 Let

$$\mu_{I}(\delta; c, \sigma) = \max\left\{1 - \frac{f^{2}}{2(1-\sigma)h^{2}}, 0\right\} \text{ if } \sigma \in [0,1) \text{ and } \delta \in [1, \delta_{\max}],$$
(36a)

$$\mu_I \in (0,1] \text{ if } \sigma = 1 \text{ and } \delta = \delta_{\max}, \tag{36b}$$

$$\mu_O\left(\delta; c, \sigma\right) = \max\left\{1 - \frac{2f^2\sigma}{h^2}, 0\right\}.$$
(37)

Under Assumptions 2 and 3, the unique pure strategy Nash equilibrium of the whereto-lobby game is: II if and only if  $\mu \in (0, \mu_I]$ , BB if and only if  $\mu \in (\mu_I, \mu_O)$ , OO if and only if  $\mu \in [\mu_O, 1]$ .



Figure 3: Illustration of Proposition 2

**Proof.** See Appendix A.3.

This proposition shows that under decentralization, depending on the values of the profit distribution, cost and greediness parameters ( $\sigma$ ,  $\delta$  and  $\mu$ , respectively), firms may end up lobbying for one public good only, either the one provided at home (when the equilibrium is II) or the one produced abroad (when the equilibrium is OO). Notice that this is in sharp contrast with the case of centralization, in which, as we showed above, both firms always end up lobbying for both public goods (the equivalent of equilibrium BB under decentralization).

Proposition 2 is illustrated in Figure 3. For a given value of  $\sigma$ , the two curves  $\mu_I$ and  $\mu_O$  (both increasing in  $\sigma$ ) with  $\mu_I < \mu_O$ ,<sup>18</sup> define the areas in the  $(\mu, \delta)$ -plane in which the Nash equilibria OO, BB and II occur. A change in  $\sigma$ , by shifting  $\mu_I$  and  $\mu_O$ , changes the size of the three areas (see the five graphs in the figure). When  $\sigma = 0$ (bottom-left graph), if the policy maker is fully benevolent, the Nash equilibrium of the where-to-lobby game is OO (since  $\mu_O = 1$ ). If instead the policy maker cares for contributions, the equilibrium is either BB or II — separated by the locus  $\mu_I$ . For  $0 < \sigma < \frac{1}{2}$  (top-left graph), as  $\sigma$  increases and approaches the value of  $\frac{1}{2}$ , both  $\mu_I$ and  $\mu_O$  shift downwards and at the same time their vertical distance shrinks. The two curves eventually overlap for  $\sigma = \frac{1}{2}$  (bottom-center graph); the equilibrium is OO if

<sup>&</sup>lt;sup>18</sup> To be more precise,  $\mu_I < \mu_O < 1$  for  $1 \le \delta < \delta_{\max}$  and  $\mu_I = \mu_O = 1$  for  $\delta = \delta_{\max}$ .

 $\mu > \mu_I = \mu_O$  and II otherwise; equilibrium BB never occurs. For  $\frac{1}{2} < \sigma < 1$  (top-right graph), as  $\sigma$  increases and approaches 1, the gap between  $\mu_O$  and  $\mu_I$  widens again, thus defining three non-empty areas corresponding to Nash equilibria OO, BB and II. Finally, when  $\sigma = 1$  (bottom-right graph), the equilibrium is II if markets are monopolized by the home firm ( $\delta = \delta_{\max}$ ), and it is either OO or BB — separated by the locus  $\mu_O$  — if each market is a duopoly ( $\delta < \delta_{\max}$ ).

As the graphs in Figure 3 show, for a given  $\sigma$ , the equilibrium OO tends to occur in the top-right area of the  $(\mu, \delta)$  plane, while II occurs in the bottom-left region. In other words, for a given  $\mu$ , firms generally lobby only at home when their market power is substantially higher at home than abroad (a high value of  $\delta$ ), whereas they generally lobby only abroad when their market power is similar in both markets. Conversely, for a given  $\delta$ , firms generally lobby only at home when politicians are rather greedy (a low value of  $\mu$ ), whereas they generally lobby only abroad when facing more benevolent policy makers (a high  $\mu$ ). This pattern hinges upon two contrasting forces. On the one hand, since the weight assigned by politicians to contributions from the home firm is lower than the one assigned to contributions from abroad, firms have an advantage in lobbying abroad. On the other hand, since a firm makes more profits at home than abroad  $(h \ge f)$ , it has greater incentives to lobby at home. These contrasting effects explain why equilibria OO and II occur respectively in the top-right and in the bottom-left regions of the  $(\mu, \delta)$  space. Unless  $\sigma = \frac{1}{2}$ , however, the OO and II regions are always "separated" by an area in which the Nash equilibrium is BB. The crucial parameter is the market structure. As we saw above, for a given  $\mu$ , firms generally lobby only abroad when their market power is balanced (low  $\delta$ ), and only at home when their interests in the foreign market are low (high  $\delta$ ). Provided that  $\sigma \neq \frac{1}{2}$ , there exist intermediate values of  $\delta$  in which firms end up lobbying in both regions, since the two above mentioned factors pointing respectively to lobbying only abroad or only at home have roughly the same force.

Finally, the graphs in Figure 3 also show that the area associated to the equilibrium OO widens as  $\sigma$  increases, while the one associated to II shrinks. The reason is simple. When  $\sigma$  is low, the equilibria of the common agency games within the cells BB, IB, OB, OI, BI, BO and IO are such that the home lobby makes high profits at home while the foreign lobby makes low profits, making II more likely to occur as a Nash equilibrium. The converse is true for high values of  $\sigma$ .

#### Lobbying equilibria and firms' profits

In the remaining part of this section we examine how the various Nash equilibria of the where-to-lobby game score in terms of firms' net profits. A full welfare analysis, that includes politicians' payoffs and consumers' surplus, is left to Section 4.3, in which centralization and decentralization are compared.

**Proposition 3** Under Assumptions 2 and 3, for a given triple  $(\sigma, \mu, \delta)$ ,

- if the Nash equilibrium of the where-to-lobby game is BB, then BB is Pareto efficient in terms of firms' net profits;
- if the Nash equilibrium is II, then II is Pareto dominated by BB;
- if the Nash equilibrium is OO, then OO is Pareto efficient if and only if  $\mu \in [\mu_{BO}, 1]$ , while it is Pareto dominated by BB if and only if  $\mu \in [\mu_O, \mu_{BO})$ , where  $\mu_O$  is defined in (37), and  $\mu_{BO} \ge \mu_O$  is defined as

$$\mu_{BO}\left(\delta;c,\sigma\right) = \frac{h^2}{h^2 + 2f^2\sigma}.\tag{38}$$

## **Proof.** See Appendix A.4. ■

Proposition 3 shows that firms can always make more profits by lobbying in both regions than by lobbying only at home. When the parameters  $(\mu, \delta, \sigma)$  are such that the equilibrium is II, the two firms end up in a "prisoner dilemma"; they would be both better off by lobbying also abroad. On the other hand, lobbying only abroad may be Pareto efficient. This is so provided that the policy maker is benevolent enough, i.e. provided that  $\mu$  is greater than the threshold  $\mu_{BO}(\delta; c, \sigma)$ . For  $\mu < \mu_{BO}$  (but greater than  $\mu_O$ , so that the equilibrium OO occurs), lobbying only abroad is dominated by lobbying in both regions (again a prisoner dilemma).

Is there any way to rule out the inefficient equilibria (in terms of firms' net profits) II and OO? In Proposition 2 we characterized the Nash equilibria of the where-tolobby game under the assumption of a uniform  $\sigma$ . With non-uniform  $\sigma^{ij}$ 's, however, lobbying under decentralization admits several other possible outcomes. Let us then consider a particular class of such cases, namely the one in which firms coordinate on a profile of the profit sharing parameters that brings about the efficient outcome BBas the Nash equilibrium of the where-to-lobby game for all  $(\mu, \delta)$  pairs.

It is quite simple to spot a set of sufficient conditions that can achieve this result. Suppose that firms coordinate on (i) the shifting parameters  $\sigma^{IB} = \sigma^{BI} = 1$  in the games IB and BI and on (ii)  $\sigma^{OB} = \sigma^{BO} = 0$  in the games OB and BO.<sup>19</sup> Also suppose that in game BB they coordinate on  $\sigma^{BB} = 0$  so as to maximize their aggregate net profits in this game. Conditions (i) and (ii) ensure that the best response to strategy B by one player is, for the other firm, to play B. Condition (i) also ensures that the best response to strategy I is B. Finally, condition (ii) ensures that the best response to strategy O is B. Hence, BB — the lobbyists preferred outcome — is the only Nash equilibrium in dominant strategies of the where-to-lobby game. Proposition 4 formalizes the argument.

**Proposition 4** If  $\sigma^{BB} = 0$ ,  $\sigma^{IB} = \sigma^{BI} = 1$  and  $\sigma^{OB} = \sigma^{BO} = 0$ , then, for all  $\mu$  and  $\delta$ , BB is the Pareto efficient Nash equilibrium in terms of firms' net profits of the where-to-lobby game under decentralization.

**Proof.** Immediate by inspection of the where-to-lobby game in Table 1. ■

Proposition 4 shows that under decentralization firms can always find a way of coordinating on lobbying in both regions and in this way reap maximal benefits from lobbying. But there are at least two caveats worth noticing. Firstly, the Nash equilibrium *BB* characterized in Proposition 4 is only one out of an infinite set. For relevant subsets of the parameters  $(\mu, \delta, \sigma^{ij})$ , the where-to-lobby game admits the inefficient outcome *II* and the possibly inefficient outcome *OO* as a Nash equilibrium. Secondly, as we show in the next section, even when the equilibrium is *BB* and firms coordinate so as to get the highest net payoffs at this equilibrium, lobbies make less profits under decentralization than under centralization and so are worse off.

## 4.3 Centralization vs Decentralization

Using the results of the previous sections, we can compare centralization and decentralization under lobbying behavior along various dimensions: social welfare, public good supply, firms' net profits, and contributions to politicians. Recall from Proposition 2 that when policy makers are fully benevolent ( $\mu = 1$ , i.e. m = 0), then under decentralization firms lobby only abroad. In this case, the equilibrium *OO* is equivalent to the one arising under centralization. In the latter regime, equilibrium contributions are nil, since firms are not able to distort the behavior of a benevolent policy maker setting the optimal level of public good provision. In the equilibrium *OO* under decentralization, by offering a contribution abroad, firms induce the benevolent local policy makers to internalize the spillover effects on foreign firms profits of public good supply, thus

<sup>&</sup>lt;sup>19</sup>Notice that the values of the profit sharing parameters  $\sigma^{OI}$  and  $\sigma^{IO}$  are irrelevant for the current argument.

realizing the same efficient equilibrium occurring under centralization. When policy makers are not benevolent ( $\mu < 1$ ), however, the two regimes are no longer equivalent.

In a centralized system, for  $\mu < 1$ , since both firms always lobby for both public goods, the resulting upward distortion in public good supply reduces social welfare. However, as formally proved in Proposition 5 below, when the equilibrium under decentralization is BB, the same distortion occurs and the supply of public goods is the same under centralization and decentralization. Again, in the latter regime the joint lobbying effort exerted by firms on both regional policy makers induces the latter to account for the regional profit-spillovers. However, when the equilibrium is such that firms lobby at most one policy maker (i.e. when the equilibrium is either II or OO), lobbying is less effective and the distortion in public good supply, and the associated welfare loss, are lower under decentralization than under centralization. Moreover, lobbies always prefer a centralized system over a decentralized one, since net profits are higher. This is obvious when the decentralized equilibrium is BB, since gross profits are the same under the two regimes whereas contributions are higher under decentralization than under centralization. Firms are also clearly better off under centralization whenever the equilibrium under decentralization is either II or OO, since in the latter case gross profits are lower while contributions, though smaller in some cases, do not allow higher net profits compared to centralization. The proposition makes these results precise.

**Proposition 5** For all lobbying equilibria under decentralization characterized in Propositions 2 and 4, firms' net profits are always higher in the corresponding lobbying equilibria under centralization. Contributions to politicians are higher under decentralization when the equilibrium is BB and, provided that  $\mu > \mu^S(\delta; c) = \frac{h^2(h^2+2f^2)}{(h^2+f^2)^2}$ , also when the equilibrium is OO; otherwise contributions are higher under centralization. Public good supply and aggregate social welfare are the same under the two regimes when the equilibrium under decentralization is BB. When the equilibrium is either II or OO, public good supply is lower, whereas aggregate social welfare is higher, under decentralization than under centralization.

#### **Proof.** See Appendix A.5. ■

The focus of our normative analysis is different from the one usually taken in former studies. We are mainly interested on the effects of lobbying on social welfare (as we define it) under different institutional arrangements. On the contrary, the literature on common agency (e.g. Bernheim and Whinston, 1986a), on one-principal many-agents models (Segal, 1999), and on games played through agents (Prat and Rustichini, 2003) defines the efficient allocations as those maximizing the sum of principal(s) and agent(s) payoffs. In these works, any third party affected by the principal-agent relationship (in our model: the consumers) is usually absent or is not explicitly considered, contrary to our analysis. This makes it difficult to compare our results with those obtained in this literature. Note, however, that if we focus on principals' (firms) and agents' (politicians) payoffs only, the equilibria in truthful strategies of our common agency game under centralization turn out to be Pareto efficient, as it is standard in the literature. Under decentralization, instead, several equilibria in our game are Pareto inefficient. For instance, if the lobbies could enforce the equilibrium BB for any  $(\mu, \delta)$ , both the firms and the politicians would be better off than in the equilibria where each firm lobbies at home only (and, under some parameters' configuration, better off even with respect to the equilibria where firms lobby only abroad). In fact, as Proposition 3 shows, firms' net profits are higher in the equilibrium BB than in the other equilibria and, for construction of the lobbying game, politicians's payoffs cannot be lower in BBthan in the other equilibria, or they would not accept the contributions. This finding is similar to Prat and Rustichini (2003) result that, in a "game played though agents", truthful equilibria may be inefficient. The source of inefficiency is however different in the two cases. In Prat and Rustichini (2003), inefficiency arises as an effect of the (possible) inexistence of pure strategy equilibria. In our case, instead, inefficiency derives from the existence of spillover effects across agents (through local public good provision) and the resulting coordination failure among principals. In this respect, our result is closer in spirit to Segal (1999), who shows that the presence of direct externalities may induce inefficient equilibria in one-principal many-agents model.

## 5 Lobbying for the market

In the previous section, we discussed the case where both firms where already operating in both markets and had a common interest in lobbying politicians to increase public good supply. We now consider a different scenario, one in which firms lobby to acquire the *right to enter* in the local markets. Again, we compare centralization and decentralization under this lobbying decision. For reasons of analytical tractability, we assume in this section that decisions about public goods (once the firms are allowed in the markets) are taken efficiently by politicians.

We study the problem by assuming the following time line. At stage 1, the government (central or regional, depending on the case) decides on the *number* of firms that are allowed to sell in the local market for good x, if one or two firms.<sup>20</sup> If both firms

 $<sup>^{20}</sup>$ The timing considered in this section is reversed with respect to that assumed in the previous one, as it is now the politician to move first and decide how many firms to allow in the market. This is done

are allowed to enter, there is clearly no need to lobby and the game goes directly to stage 4. On the other hand, if the government decides to allow for one entrant only at stage 1, at stage 2 each firm competing for the market makes a credible commitment to offer the politician a positive contribution if she is the only firm allowed to enter in the market for good x. At stage 3, the politician, knowing the offers made by firms at stage 2, assigns the monopoly right to the firm that guarantees her the highest payoff (i.e. the weighted average of social welfare and lobbies' contributions) and cashes the relevant contribution. At stage 4, the government chooses public good supply by maximizing social welfare. Finally, at stage 5, market equilibrium is determined along the lines of Section 2. The model is solved by backward induction.

Notice that the more complex structure of this case allows us to consider a richer institutional setting. We can still consider a case of *full centralization*, when the central government chooses both the number of firms entering each regional market and local public good supplies, and a case of *full decentralization*, where each regional government chooses both the number of firms entering its market and public good supply. But we may also have a case of *split competencies*, where the central government establishes the number of firms that are allowed to operate in each regional market but regional public good supply is decided at the regional level. This case mimics the situation in many countries (and the EU), where regulation policy is centralized but decisions about local public goods (infrastructures in our case) are decentralized. We ask if lobbying may provide a rationale for these arrangements.

To investigate these three cases, we need first to compute market equilibrium and welfare under monopoly (stage 5), thus extending the duopoly analysis already provided in Section 2. Letting

$$H = \frac{1-c}{2} \quad \text{and} \quad F = \frac{1-\delta c}{2},\tag{39}$$

we obtain, by standard profit maximization, that when the regional markets are monopolized the equilibrium quantities are  $x_a^* = Hg_a$  and  $x_b^* = Hg_b$  ( $x_a^* = Fg_a$  and  $x_b^* = Fg_b$ ) if it is the home (foreign) firm that supplies the market. The corresponding equilibrium profits are  $\Pi_{\alpha}^* = H^2g_a$  and  $\Pi_{\beta}^* = H^2g_b$  ( $\Pi_{\alpha}^* = F^2g_b$  and  $\Pi_{\beta}^* = F^2g_a$ ) when the home (foreign) firm supplies the market.

Focusing again on region a, and depending on which firm operates in each region,

for analytical simplicity: the corresponding game with lobbies moving first, in fact, turns out to be too complex to allow for analytic solutions. However, as it will become clear in the next pages, since local politicians care for the profits of their home firms only, the main results of this section should remain unchanged also under the original timing, regardless of the fact that with the new timing all bargaining power rests with the politician.

social welfare is

$$W_a^{H_a H_b} = \frac{3H^2 g_a}{2} - \frac{g_a^2}{4} + \bar{y},\tag{40}$$

$$W_a^{F_a F_b} = \frac{F^2 g_a + 2F^2 g_b}{2} - \frac{g_a^2}{4} + \bar{y}, \tag{41}$$

$$W_a^{H_a F_b} = \frac{3H^2 g_a + 2F^2 g_b}{2} - \frac{g_a^2}{4} + \bar{y}, \tag{42}$$

$$W_a^{F_a H_b} = \frac{F^2 g_a}{2} - \frac{g_a^2}{4} + \bar{y},\tag{43}$$

where the apex  $H_aH_b$  (resp.  $F_aF_b$ ) denotes that home (resp. foreign) firms are monopolists in both regions, and  $H_aF_b$  (resp.  $F_aH_b$ ) that firm  $\alpha$  (resp.  $\beta$ ) is a monopolist in both regions. We begin the analysis with the full centralization case.

## 5.1 Full centralization

Invoking symmetry, we only consider the case in which the central government opts at stage 1 for the same policy, one or two firms, in both regions. Suppose first that the government allows for both firms supplying both regional markets. This case was already studied in Section 3, where we described the equilibrium without lobbying. Substituting the optimal public good provision given in (12) into (11), the politician's value function when both firms are allowed to enter the market is then

$$\hat{V}^{hf} = \mu \frac{[(h+f)^2 + 2(h^2 + f^2)]^2}{2} + 2\mu \bar{y}.$$
(44)

Consider next the case in which only one firm is allowed to enter the regional markets. The government holds simultaneously an auction for each market, and firms have an incentive to compete for it, offering contributions to the government. Let  $S_{\rho}^{H}$  and  $S_{\rho}^{F}$  be the contribution offered by firm  $\rho$  for serving the home and the foreign market, respectively. The following Lemma summarizes the outcome of firms' competition for the market.

**Lemma 1** Under full centralization, if only one firm is allowed to enter the regional markets, then each firm gets the home market by paying the contribution

$$\hat{S}^{H}_{\rho} = \max\left\{\hat{T}^{H}, 0\right\}, \text{ where } \hat{T}^{H} = -\frac{9\mu(H^{4} - F^{4})}{4(1-\mu)} + 3F^{4}.$$
 (45)

The corresponding politician's value function is

$$\hat{V}^{H} = \mu \frac{9H^{4}}{2} + 2(1-\mu)\hat{S}^{H}_{\rho} + 2\mu\bar{y}.$$
(46)

#### **Proof.** See Appendix B.

The intuition is simple. A local monopoly is always more profitable than a foreign one, since the home firm has a cost advantage over the foreign one  $(H \ge F)$ , and the optimal public good supply is higher when the home firm serves the market (for the same reason). Hence, each firm wins the home market by outbidding the foreign firm, whose offer  $\hat{S}_{\rho}^{F}$  at most equals the profits it would make by serving the foreign market in a monopolistic regime,  $3F^{4}$ . Notice, however, from (45) that the home firm does not need to offer that much, and in some cases it does not even need to make a positive offer, to win the market. The reason is that if the foreign firm gets the market, then a welfare loss is observed compared to a home-monopoly. Thus, in order to win the market, the home firm can always offer the politician a lower contribution than the one offered by the foreign firm. Quite intuitively, the higher are  $\mu$  and  $\delta$  the more likely is that the home firm does not need to make a positive offer to win the market.

By comparing (44) and (46), we can then characterize the central government's choice in stage 1.

**Proposition 6** Under full centralization, for  $\delta \in [1, \delta_1]$ ,  $\delta_1(c) = \frac{5+17c}{22c}$ , there exists a  $\mu_1(\delta; c)$ , decreasing in  $\delta$ , such that for all  $\mu \leq \mu_1$  one firm only is allowed to enter each regional market; by Lemma 1, the home firm obtains a monopoly upon the payment of a contribution. For  $\delta \in [1, \delta_1]$  and  $\mu > \mu_1$  both firms are allowed into both regional markets. For  $\delta \in (\delta_1, \delta_{\max}]$ , one firm only is allowed to enter each regional market for all  $\mu$  and therefore the home firm gets a monopoly. As for contributions, there exists a  $\mu_2(\delta; c)$ , decreasing in  $\delta$ , such that the home firm pays a contribution for all  $\mu < \mu_2$  and no contribution otherwise.

#### **Proof.** See Appendix B.

Figure 4-a. illustrates the Proposition. For  $\delta \leq \delta_1$  and  $\mu > \mu_1$  the policy maker opts for a duopoly in both markets  $(hf_ahf_b)$ . In all other cases, she opts for a monopoly and, given the results in Lemma 1, each firm wins its home market  $(H_aH_b)$ . In this latter case, positive contributions  $(\hat{S}_{\rho}^H > 0)$  are paid if and only if  $\mu$  is below a given threshold  $(\mu_1 \text{ or } \mu_2$ , depending on the value of  $\delta$ ); otherwise the home firm does not need to offer a contribution to gain access to the monopolized market.

To understand the intuition behind these results, suppose first that the politician simply maximizes social welfare (i.e.  $\mu = 1$ ). The proposition then shows that there exists a threshold level of the cost advantage for the home firm,  $\delta_1$ , such that for  $\delta < \delta_1$  $(\delta \ge \delta_1)$ , social welfare is higher (lower) under a duopoly than under a monopoly. Hence, the fully benevolent politician simply lets both firms enter both markets in the former case and only the home firm in the latter one. If instead  $\mu < 1$ , the politician



Figure 4: Lobbying for the market

faces a trade-off when  $\delta < \delta_1$ . By creating a monopoly, she gets a contribution from the home firm winning the contest for the market, but at the cost of the monopoly welfare loss; however, if she lets both firms in, she avoids this welfare cost but does not get any contribution. This explains why, for  $\delta < \delta_1$ , a sufficiently benevolent policy maker — one with preferences  $\mu > \mu_1$  — makes the efficient choice, while a politician who is greedier  $(\mu \leq \mu_1)$  prefers a monopoly by home firms in each regional market. This trade off is absent when  $\delta \geq \delta_1$ , since social welfare is however higher under a home monopoly than under a duopoly. Hence, the politician always allows one firm only in each market, no matter her degree of benevolence. The latter only bears on whether contributions are paid to the central politician. If  $\mu > \mu_2$ , i.e. if the politician is sufficiently benevolent, then home firms would not need to bribe the politician in order to win the local monopoly, even though foreign firms made a positive offer. Instead, if the politician is greedy ( $\mu \leq \mu_2$ ), the home firm must offer a contribution to outbid the offer made by the foreign firm. Recalling that lobbies' contributions are pure transfers and that when lobbying is for the market there are no distortions in public goods supply, we can conclude that a loss in social welfare occurs if and only if lobbying induces the central government to opt for local monopolies whenever a benevolent social planner would have opted for local duopolies. Formally:

**Corollary 1** Under full centralization lobbying causes a welfare loss if and only if  $\delta \in [1, \delta_1)$  and  $\mu \in (0, \mu_1)$ .

## 5.2 Split competencies

Consider next the case in which the central government chooses how many firms enter each market, but the regional governments choose public good supply. Since what differentiates split competencies and the fully centralized regimes is only the equilibrium level of public goods supply, we can directly follow the above logic to prove:

**Proposition 7** Under split competencies, for  $\delta \in [1, \delta_2]$ ,  $\delta_2(c) < \delta_1(c)$  for all  $c \in (0, 1)$ , there exists a  $\mu_3(\delta; c)$  such that for all  $\mu \leq \mu_3$  one firm only is allowed to enter each regional market, and therefore the home firm obtains a monopoly upon the payment of a contribution; otherwise both firms are allowed into both regional markets. For  $\delta \in (\delta_2, \delta_{\max}]$  one only firm is allowed to enter each regional market for all  $\mu$ , and hence the home firm gets a monopoly. As for contributions, there exists a  $\mu_4(\delta; c)$  such that the firm pays a contribution for all  $\mu < \mu_4$  and no contribution otherwise.

Split competencies and full centralization are compared in Figure 4-b. Notice that the area in which each firm obtains a monopoly at home upon the payment of a contribution is certainly *smaller* under split competencies, since the curves  $\mu_3$  and  $\mu_4$  for the latter case lie below the respective curves  $\mu_1$  and  $\mu_2$  for centralization. Hence, lobbying for the market is less effective under split competencies than under centralization.

However, the comparison in terms of social welfare depends on parameters. As  $\delta_2 < \delta_1$ , there is an area under split competencies — defined by  $\delta \in (\delta_2, \delta_1)$  and  $\mu > \mu_1$ — in which even a fully benevolent central politician ( $\mu = 1$ ) would opt for a monopoly by the home firm instead of the more efficient duopoly. This is so because under split competencies public good provision is decided at the local level and as shown above (in Section 3) local public goods are underprovided by local governments in local duopolies. Hence, allowing for a single home producer by the center is a way to partly counteract this inefficiency at local level. On the other hand, split competencies is more efficient than centralization for  $\delta < \delta_2$ , as the set in which two firms are allowed in both markets (the efficient choice) is larger under split competencies than under centralization, since  $\mu_3 < \mu_1$ . This is again due to the fact that local governments do not consider foreign firms' profits as part of the (local) social welfare. In fact, in the event of a foreign monopoly, a local government undersupplies the public good compared to a central government. This means that under split competencies the home firms offer the politician a smaller contribution to outbid the foreign competitor, which explains why the central government is more willing to let two firms enter the market. Thus, lobbying is more effective under full centralization.

#### 5.3 Full decentralization

We finally consider the case of full decentralization, in which regional governments (simultaneously) choose first the number of firms that are allowed to enter their market,

		region b	
		two firms	one firm
r	two	$\hat{V}_a^{hf} = \mu \frac{[(h+f)^2 + 2h^2][(h+f)^2 + 2(h^2 + f^2)]}{4}$	$\hat{V}_a^{hf_aH_b} = \mu \frac{[(h+f)^2 + 2h^2]^2}{4}$
е	firms		
g.	one	$\hat{V}_a^{hf_aH_b} =$	$\hat{V}_a^{H_aH_b} =$
a	firm	$= \begin{cases} \frac{4f^{2}[(h+f)^{2}+2h^{2}]\mu+(4+\mu)F^{4}}{4} & \text{if } \mu < \mu_{5}(\delta;c), \\ \mu \frac{9H^{4}+4f^{2}[(h+f)^{2}+2h^{2}]}{4} & \text{otherwise.} \end{cases}$	$= \begin{cases} \frac{F^4}{4} (4+\mu) & \text{if } \mu < \mu_5(\delta;c), \\ \frac{9H^4}{4}\mu & \text{otherwise.} \end{cases}$

Table 2: Politicians' value functions under full decentralization

and then public good supply. The choice on the number of firms gives rise to a  $2 \times 2$  normal form game between regional policy makers. Whenever one firm only is allowed to supply a regional market, firms compete by bribing the regional policy maker. For any strategy pair, Lemma 2 establishes the outcome of firms' competition for the market and regional payoffs, shown in Table 2, in terms of the maximum value of politicians' objective functions.<sup>21</sup>

**Lemma 2** Under full decentralization, whenever a region allows for one firm only to serve its local market, then it is the home firm to gain access to the market, paying the contribution

$$\hat{S}_{\rho}^{H} = \max\left\{-\frac{\mu(9H^{4} - F^{4})}{4(1-\mu)} + \frac{F^{4}}{1-\mu}, 0\right\}$$
(47)

to the politician.

Depending on the number of firms allowed into each regional markets, politicians' value functions are those shown in Table 2.

#### **Proof.** See Appendix B. ■

As in the previous regimes and for the same reasons, it is always the home firm to gain a monopoly in its market when competing with the foreign firm. From (47) it is immediate to see that  $\hat{S}_{\rho}^{H} > 0$  if and only if

$$\mu < \mu_5(\delta; c) = \frac{4F^4}{9H^4 - F^4}.$$
(48)

Regional politicians choose the number of firms in the market by playing the normal form game given in Table 2. The solution of such a policy game is given in the following proposition:

<sup>&</sup>lt;sup>21</sup>Since the game is symmetric, the Table shows only the payoffs of region *a*'s politician. Also, to save space, regional social welfare is net of the endowment  $\bar{y}$ .

**Proposition 8** Under full decentralization, it is a dominant strategy for both regional policy makers to admit one firm only in their market for all  $\mu \in (0, 1]$  and  $\delta \in [1, \delta_{\max}]$ . Hence, by Lemma 2 the home firm gets a local monopoly upon the payment of a positive contribution for  $\mu < \mu_5(\delta; c)$  and nothing otherwise.

#### **Proof.** See Appendix B.

Proposition 8 shows that lobbying for the market is most effective under full decentralization, with the home firms always gaining a local monopoly in their regional market. When  $\delta < \delta_1$ , although a duopoly would be the efficient solution in both regions, markets turn out to be fully monopolized no matter the value of  $\mu$ . This means that in the case of lobbying for the market full decentralization is the least efficient of the three regimes. Moreover, one can show that the Nash equilibrium (one-firm, onefirm) of the game in Table 2 is also Pareto inefficient in terms of politicians' aggregate value functions for all  $\delta < \delta_2$ . The difference between full decentralization and split competencies is that, while under the former regime regional policy makers end up in a prisoner dilemma, under the latter regime this outcome does not occur because it is the central policy maker that directly chooses the highest aggregate payoff along the diagonal cells of the game in Table 2.

# 6 Concluding remarks

Is decentralization more conductive to lobbying behavior? This paper offers a simple answer to this important question. When interests of local lobbies are aligned, as in our "in the market case", then decentralization is better than centralization because the former institutional setting makes it more difficult to coordinate behavior across lobbies, and in any case lobbies have to pay larger bribes to politicians to induce them to internalize profits spillover effects. Vice-versa, when interests of local lobbies are in conflict, as in our "for the market case", then centralization is better than decentralization because local governments are more easily captured by local interests. This result strongly suggests that in deciding whether a given function should be decentralized (resp. centralized) in presence of significant lobbying behavior, one should also consider how the interests of local lobbies are positioned with respect to that particular function.

For instance, taking again the EU example cited in the Introduction, one notes that in fields such as consumer and environment protection, foreign and domestic producers would have the same interest to lobby for low consumers' protection if these policies were decided at the EU level. Of course, they would do the same if the policies remained at local level, but then each country would have no interest to internalize the effects of these policies on the profits of foreign firms, leading to lower distortions. *Coeteris paribus*, our argument would then suggest to decentralize these functions. Vice-versa, in regulatory fields such as production subsidies to national producers, protection of market share of incumbents and "national champions", national lobbies have conflicting interests, and centralization at the EU level would force the policy maker to take into account also the interests hurt by protection policy. Hence, *coeteris paribus*, our argument would suggest to centralize these functions.

The paper also offers some other interesting insights. As we saw in the "for the market" case, there are situations in which the best institutional structure under lobbying is one where competencies are split between different levels of government, with the central level taking decisions about regulation policy and the lower level taking decisions about local public good supply. Indeed, we often observe in many countries that different levels of government interact, with different roles, on the same function. This is in contrast with the prescriptions of traditional normative fiscal federalism theory, which usually suggests a one-to-one assignment of functions to different levels of government. Asking if this division of tasks performs some efficiency functions, such as greater resilience to interest groups, would be an interesting avenue for further research.

Finally, there are many respects in which the above analysis calls for extensions. Our modelling of lobbying behavior is rough, as it refers to the first generation of lobbying models, which are, as we already remarked, a reduced form of a far more complex political behavior, involving elections and legislatures. Allowing for a more complex institutional structure (along the lines, for example, of Persson and Tabellini, 2000, ch. 7, Mitra, 1999, Besley and Coate, 2001, Felli and Merlo, 2001) may highlight other channels of interaction between local interests and local policies which we have not considered here. Moreover, lobbying is not necessarily a "bad", as we have assumed here. For instance, it may provide useful information to politicians and citizens. Since better information on policies and politicians is often quoted as one of the main advantage of decentralization (e.g. Bennedsen and Feldmann, 2002 and 2004, Besley and Smart, 2003b; Bordignon *et al.*, 2004), discussing the link between informational lobbying and decentralization may offer further useful insights.

# A Appendix: Lobbying for public good provision

## A.1 Centralization

When both firms lobby the policy maker, from the first order conditions for maximizing (17),

$$\mu \frac{\partial \mathcal{W}}{\partial g_r} + (1-\mu)(h^2 + f^2) = 0, \tag{A.1}$$

we obtain  $\tilde{g}^C$  in (18) for both  $g_a$  and  $g_b$ . In deriving the first order condition (A.1), we ignore the non-negativity constraint on contributions, by letting  $S_{\rho} = h^2 g_r + f^2 g_{-r} - \pi_{\rho}$  into the objective function (17), and then checking non-negativity *ex post* in the computed equilibrium. Substituting  $\tilde{g}^C$  into (17) we get the politician's welfare,  $V^C(\tilde{g}_a^C, \tilde{g}_b^C, \pi_\alpha, \pi_\beta)$ , as a function of firms net profits. To compute the equilibrium net profits (and contributions), we derive the firms' best response functions. To get the best response function of firm  $\beta$ , we solve first the problem in which firm  $\alpha$  is lobbying while  $\beta$  is not. In this case, the policy maker maximizes  $V_{-\beta}^C(g_a, g_b, \pi_\alpha) = \mu \mathcal{W} + (1 - \mu)S_{\alpha}$ . From the corresponding first order conditions:

$$\mu \frac{\partial \mathcal{W}}{\partial g_a} + (1-\mu)h^2 = 0, \qquad \mu \frac{\partial \mathcal{W}}{\partial g_b} + (1-\mu)f^2 = 0,$$

we obtain the optimal public good supplies:

$$\tilde{g}_{a(-\beta)}^{C} = \hat{g}^{C} + 2mh^{2}, \qquad \tilde{g}_{b(-\beta)}^{C} = \hat{g}^{C} + 2mf^{2}.$$

For any given  $\pi_{\alpha}$ , the maximum amount of profits that firm  $\beta$  can make is constrained by the politician's participation constraint, which gives rise to the following conditions:

$$V^{C}(\tilde{g}_{a}^{C}, \tilde{g}_{b}^{C}, \pi_{\alpha}, \pi_{\beta}) = V^{C}_{-\beta}(\tilde{g}_{a(-\beta)}^{C}, \tilde{g}_{b(-\beta)}^{C}, \pi_{\alpha}), \quad \text{if } \tilde{V}^{C}_{-\beta} \ge \mu \mathcal{W}(\hat{g}_{a}^{C}, \hat{g}_{b}^{C}), \tag{A.2a}$$

$$V^C(\tilde{g}_a^C, \tilde{g}_b^C, \pi_\alpha, \pi_\beta) = \mu \mathcal{W}(\hat{g}_a^C, \hat{g}_b^C), \quad \text{otherwise.}$$
(A.2b)

Given  $\pi_{\alpha}$ , if the policy maker is better off under lobbying by firm  $\alpha$  than under no lobbying, i.e. if  $\tilde{V}_{-\beta}^{C} \geq \mu \mathcal{W}(\hat{g}_{a}^{C}, \hat{g}_{b}^{C})$ , then Eq. (A.2a) says that firm  $\beta$  can increase its profits (and correspondingly reduce contributions) up to the point in which the policy maker is indifferent between being lobbed by both firms and being lobbed only by firm  $\alpha$ . On the contrary, if the policy maker is better off under no lobbying than under lobbying by firm  $\alpha$ , then (A.2b) says that firm  $\beta$  can increase its profits up to the point in which the policy maker is indifferent between being lobbed by both firms and not being lobbed.

It is immediate to show that condition  $\tilde{V}_{-\beta}^C \ge \mu \mathcal{W}(\hat{g}_a^C, \hat{g}_b^C)$  in (A.2a) can be written as  $\pi_{\alpha} \le \pi_1 \equiv \hat{\pi}^C + m(h^4 + f^4)$ . Thus, solving the equations in (A.2a) and (A.2b) for  $\pi_{\beta}$ , we get the best response function of firm  $\beta$  as

$$\pi_{\beta}^{*}(\pi_{\alpha}) = \begin{cases} \pi_{2} & \text{if } \pi_{\alpha} \leq \pi_{1}, \\ \hat{\pi}^{C} + \pi_{3} - \pi_{\alpha} & \text{if } \pi_{1} < \pi_{\alpha} \leq \pi_{3}, \\ \hat{\pi}^{C} & \text{if } \pi_{\alpha} > \pi_{3}, \end{cases}$$
(A.3)

where  $\pi_2 \equiv \pi_1 + 4mh^2 f^2$  and  $\pi_3 \equiv \pi_2 + m(h^4 + f^4)$ . The graph of  $\pi^*_\beta(\pi_\alpha)$  is shown in Figure 1 in the text as the solid curve. For  $\pi_\alpha \leq \pi_1$ , the best response of firm  $\beta$  is to offer the policy maker

a contribution schedule such that  $\pi_{\beta} = \pi_2$ , since this is the minimum amount of contributions (maximum amount of profits) that keeps the policy maker on its participation constraint. For  $\pi_{\alpha} > \pi_1$ , as firm  $\alpha$  reduces its contributions, the best response of  $\beta$  is to correspondingly increase its contributions (the slope of the best response function is -1). However, firm  $\beta$  is not willing to see its profits to go below  $\hat{\pi}^C$ , the amount it would earn without lobbying; hence the best response function is flat at  $\pi_{\beta}^* = \hat{\pi}^C$  for  $\pi_{\alpha} > \pi_3$ .

By symmetry, we get the best response function,  $\pi^*_{\alpha}(\pi_{\beta})$ , of firm  $\alpha$ , also shown in Figure 1 as the dotted curve. It is then immediate to see that the lobbying game admits a set of Nash equilibria, defined by the profit pairs  $(\pi_{\alpha}, \pi_{\beta})$  such that

$$\pi_{\beta} = \hat{\pi}^C + \pi_3 - \pi_{\alpha}, \quad \pi_1 \le \pi_{\alpha} \le \pi_2. \tag{A.4}$$

It is immediate to check that contributions are non-negative in all Nash equilibria, ranging from the minimum level of  $s_{\rho} = m(h^4 + f^4)$ , when firm  $\rho$  earns the highest amount of profits  $\pi_{\rho} = \hat{\pi}^C + \pi_3 - \pi_1$ , to the maximum level of  $s_{\rho} = m(h^4 + f^4) + 4mh^2f^2$ , when firm  $\rho$  earns the lowest amount of profits  $\pi_{\rho} = \hat{\pi}^C + \pi_3 - \pi_2$ . Finally, setting  $\pi_{\beta} = \pi_{\alpha}$ , we obtain from (A.4) the symmetric Nash equilibrium  $\tilde{\pi}^C = \tilde{\pi}^C_{\alpha} = \tilde{\pi}^C_{\beta}$  shown in (20), and by substituting (18) and (20) into (16a)–(16b), we get the contributions  $\tilde{s}^C = \tilde{s}^C_{\alpha} = \tilde{s}^C_{\beta}$  in (21).

## A.2 Decentralization: derivation of the where-to-lobby game

We solve the lobby game for each strategy pair occurring under decentralization, ignoring the non-negativity constraint on contributions, letting  $S_{\alpha a} = h^2 g_a - \pi_{\alpha a}$ ,  $S_{\beta a} = f^2 g_a - \pi_{\beta a}$ ,  $S_{\alpha b} = f^2 g_b - \pi_{\alpha b}$  and  $S_{\beta b} = h^2 g_b - \pi_{\beta b}$ . We check *ex post* that equilibrium contributions are non-negative.  $V_r^{ij}$  denotes the preferences of policy maker  $r, r \in \{a, b\}$ , when firms  $\alpha$  and  $\beta$ are choosing action *i* and *j*, respectively,  $i, j \in \{B, I, O, N\}$ . The results of the analysis are summarized in Table A.1 (equilibrium profits) and Table A.2 (equilibrium contributions). To ease the derivation of the results, we define the problems solved by the regional policy makers in the various lobbying games (games *BB*, *II* and *OO* are defined in Section 4.2).

$$\tilde{g}_{a}^{BI} = \underset{g_{a}}{\operatorname{argmax}} V_{a}^{BI}(g_{a}, g_{b}, \pi_{\alpha a}, \pi_{\alpha b}),$$
(A.5a)
$$V_{a}^{BI}(.) \equiv \mu(W_{a} - S_{\alpha b}) + (1 - \mu)S_{\alpha a}, \quad \tilde{V}_{a}^{BI} \equiv V_{a}^{BI}(\tilde{g}_{a}^{BI}, \tilde{g}_{b}^{BI}, \pi_{\alpha a}, \pi_{\alpha b}),$$

$$\tilde{g}_{b}^{BI} = \underset{g_{b}}{\operatorname{argmax}} V_{b}^{BI}(g_{a}, g_{b}, \pi_{\beta b}, \pi_{\alpha b}),$$
(A.5b)
$$V_{b}^{BI}(.) \equiv \mu(W_{b} + S_{\alpha b}) + (1 - \mu)(S_{\alpha b} + S_{\beta b}), \quad \tilde{V}_{b}^{BI} \equiv V_{b}^{BI}(\tilde{g}_{a}^{BI}, \tilde{g}_{b}^{BI}, \pi_{\beta b}, \pi_{\alpha b}).$$

$$\tilde{g}_{a}^{OI} = \underset{g_{a}}{\operatorname{argmax}} V_{a}^{OI}(g_{a}, g_{b}, \pi_{\alpha b}),$$
(A.6a)
$$V_{a}^{OI}(.) \equiv \mu(W_{a} - S_{\alpha b}), \quad \tilde{V}_{a}^{OI} \equiv V_{a}^{OI}(\tilde{g}_{a}^{OI}, \tilde{g}_{b}^{OI}, \pi_{\alpha b})$$

$$\tilde{g}_{b}^{OI} = \underset{g_{b}}{\operatorname{argmax}} \quad V_{b}^{OI}(g_{a}, g_{b}, \pi_{\beta b}, \pi_{\alpha b}),$$
(A.6b)
$$V_{b}^{OI}(.) \equiv \mu(W_{b} + S_{\alpha b}) + (1 - \mu)(S_{\alpha b} + S_{\beta b}), \quad \tilde{V}_{b}^{OI} \equiv V_{b}^{OI}(\tilde{g}_{a}^{OI}, \tilde{g}_{b}^{OI}, \pi_{\beta b}, \pi_{\alpha b}).$$

		Profit at home	Profit abroad
		$h^2 \hat{g}^D +$	$f^2 \hat{g}^D +$
firm $\alpha$	B	$mh^4 + 2(1+m)h^2f^2(1-\sigma^{BB})$	$(1+m)f^4 + 2mh^2 f^2 \sigma^{BB}$
firm $\beta$	B	$mh^4 + 2(1+m)h^2f^2(1-\sigma^{BB})$	$(1+m)f^4 + 2mh^2 f^2 \sigma^{BB}$
firm $\alpha$	Ι	$mh^4 + 2(1+m)h^2f^2(1-\sigma^{IB})$	$2mh^2f^2$
firm $\beta$	B	$mh^4$	$(1+m)f^4 + 2mh^2 f^2 \sigma^{IB}$
firm $\alpha$	0	$2(1+m)h^2f^2$	$(1+m)f^4 + 2mh^2 f^2 \sigma^{OB}$
firm $\beta$	B	$mh^4 + 2(1+m)h^2f^2(1-\sigma^{OB})$	$(1+m)f^4$
firm $\alpha$	N	$2(1+m)h^2f^2$	$2mh^2f^2$
firm $\beta$	B	$mh^4$	$(1+m)f^4$
firm $\alpha$	Ι	$mh^4$	$2mh^2f^2$
firm $\beta$	Ι	$mh^4$	$2mh^2f^2$
firm $\alpha$	0	0	$(1+m)f^4 + 2mh^2 f^2 \sigma^{OI}$
firm $\beta$	Ι	$mh^4 + 2(1+m)h^2f^2(1-\sigma^{OI})$	0
firm $\alpha$	N	0	$2mh^2f^2$
firm $\beta$	Ι	$mh^4$	0
firm $\alpha$	0	$2(1+m)h^2f^2$	$(1+m)f^4$
firm $\beta$	0	$2(1+m)h^2f^2$	$(1+m)f^4$
firm $\alpha$	N	$2(1+m)h^2f^2$	0
firm $\beta$	0	0	$(1+m)f^4$

Table A.1: Firms net profits under decentralization

$$\tilde{g}_{a}^{OB} = \underset{g_{a}}{\operatorname{argmax}} V_{a}^{OB}(g_{a}, g_{b}, \pi_{\beta a}, \pi_{\alpha b}),$$
(A.7a)
$$V_{a}^{OB}(.) \equiv \mu(W_{a} - S_{\alpha b} + S_{\beta a}) + (1 - \mu)S_{\beta a}, \quad \tilde{V}_{a}^{OB} \equiv V_{a}^{OB}(\tilde{g}_{a}^{OB}, \tilde{g}_{b}^{OB}, \pi_{\beta a}, \pi_{\alpha b})$$

$$\tilde{g}_{b}^{OB} = \underset{g_{b}}{\operatorname{argmax}} V_{b}^{OB}(g_{a}, g_{b}, \pi_{\beta a}, \pi_{\alpha b}),$$
(A.7b)
$$V_{a}^{OB}(.) \equiv \mu(W_{a} - S_{a} + S_{a}) + (1 - \mu)(S_{a} + S_{a}), \quad \tilde{V}_{a}^{OB} \equiv V_{a}^{OB}(\tilde{g}_{a}^{OB}, \tilde{g}_{b}^{OB}, \pi_{\beta a}, \pi_{\alpha b}),$$
(A.7b)

$$V_b^{OB}(.) \equiv \mu(W_b - S_{\beta a} + S_{\alpha b}) + (1 - \mu)(S_{\alpha b} + S_{\beta b}), \ \tilde{V}_b^{OB} \equiv V_b^{OB}(\tilde{g}_a^{OB}, \tilde{g}_b^{OB}, \pi_{\beta b}, \pi_{\beta a}, \pi_{\alpha b}).$$

$$\begin{split} \tilde{g}_{a}^{NI} &= \underset{g_{a}}{\operatorname{argmax}} \ V_{a}^{NI}(g_{a},g_{b}), \end{split}$$
(A.8a)  

$$\begin{split} V_{a}^{NI}(.) &\equiv \mu W_{a}, \ \tilde{V}_{a}^{NI} \equiv V_{a}^{NI}(\tilde{g}_{a}^{NI},\tilde{g}_{b}^{NI}) \\ \tilde{g}_{b}^{NI} &= \underset{g_{b}}{\operatorname{argmax}} \ V_{b}^{NI}(g_{a},g_{b},\pi_{\beta b}), \end{aligned}$$
(A.8b)  

$$\begin{split} V_{b}^{NI}(.) &\equiv \mu W_{b} + (1-\mu)S_{\beta b}, \ \tilde{V}_{b}^{NI} \equiv V_{b}^{NI}(\tilde{g}_{a}^{NI},\tilde{g}_{b}^{NI},\pi_{\beta b}). \end{split}$$

		Contributions at home	Contributions abroad
firm $\alpha$	B	$mh^4 + 2(1+m)h^2f^2\sigma^{BB}$	$(1+m)f^4 + 2mh^2f^2(1-\sigma^{BB})$
firm $\beta$	B	$mh^4 + 2(1+m)h^2f^2\sigma^{BB}$	$(1+m)f^4 + 2mh^2f^2(1-\sigma^{BB})$
firm $\beta$	Ι	$mh^4 + 2(1+m)h^2 f^2 \sigma^{IB}$	
firm $\alpha$	B	$mh^4$	$(1+m)f^4 + 2mh^2f^2(1-\sigma^{IB})$
firm $\beta$	0		$(1+m)f^4 + 2mh^2f^2(1-\sigma^{OB})$
firm $\alpha$	B	$mh^4 + 2(1+m)h^2 f^2 \sigma^{OB}$	$(1+m)f^4$
firm $\beta$	N		
firm $\alpha$	B	$mh^4$	$(1+m)f^4$
firm $\beta$	Ι	$mh^4$	
firm $\alpha$	Ι	$mh^4$	—
firm $\beta$	0		$(1+m)f^4 + 2mh^2f^2(1-\sigma^{OI})$
firm $\alpha$	Ι	$mh^4 + 2(1+m)h^2 f^2 \sigma^{OI}$	_
firm $\beta$	N		
firm $\alpha$	Ι	$mh^4$	_
firm $\beta$	0		$(1+m)f^4$
firm $\alpha$	0		$(1+m)f^4$
firm $\beta$	N		
firm $\alpha$	0		$(1+m)f^4$

Table A.2: Firms contributions under decentralization

$$\begin{split} \tilde{g}_{a}^{NB} &= \underset{g_{a}}{\operatorname{argmax}} V_{a}^{NB}(g_{a}, g_{b}, \pi_{\beta a}), \\ (A.9a) \\ V_{a}^{NB}(.) &\equiv \mu(W_{a} + S_{\beta a}) + (1 - \mu)S_{\beta a}, \ \tilde{V}_{a}^{NB} &\equiv V_{a}^{NB}(\tilde{g}_{a}^{NB}, \tilde{g}_{b}^{NB}, \pi_{\beta a}) \\ \tilde{g}_{b}^{NB} &= \underset{g_{b}}{\operatorname{argmax}} V_{b}^{NB}(g_{a}, g_{b}, \pi_{\beta b}, \pi_{\beta a}), \\ V_{b}^{NB}(.) &\equiv \mu(W_{b} - S_{\beta a}) + (1 - \mu)S_{\beta b}, \ \tilde{V}_{b}^{NB} &\equiv V_{b}^{NB}(\tilde{g}_{a}^{NB}, \tilde{g}_{b}^{NB}, \pi_{\beta b}, \pi_{\beta a}). \\ \tilde{g}_{a}^{NO} &= \underset{g_{a}}{\operatorname{argmax}} V_{a}^{NO}(g_{a}, g_{b}, \pi_{\beta a}), \\ V_{a}^{NO}(.) &\equiv \mu(W_{a} + S_{\beta a}) + (1 - \mu)S_{\beta a}, \ \tilde{V}_{a}^{NO} &\equiv V_{a}^{NO}(\tilde{g}_{a}^{NO}, \tilde{g}_{b}^{NO}, \pi_{\beta a}) \end{split}$$
(A.9a)

$$\begin{aligned}
\bar{y}_{a}^{NO} &= \mu(W_{a} + S_{\beta a}) + (1 - \mu)S_{\beta a}, \ v_{a}^{NO} &\equiv v_{a}^{NO}(g_{a}^{NO}, g_{b}^{NO}, \pi_{\beta a}) \\
\tilde{g}_{b}^{NO} &= \underset{g_{b}}{\operatorname{argmax}} \ V_{b}^{NO}(g_{a}^{NO}, g_{b}^{NO}, \pi_{\beta a}), \\
V_{b}^{NO}(.) &\equiv \mu(W_{b} - S_{\beta a}), \ \tilde{V}_{b}^{NO} &\equiv V_{b}^{NO}(\tilde{g}_{a}^{NO}, \tilde{g}_{b}^{NO}, \pi_{\beta a}).
\end{aligned}$$
(A.10b)

#### Both firms lobbying both regions (BB)

When both firms lobby both regions, policy makers' solve problems (22a)–(22b) in Section 4.2, obtaining, from the corresponding first order conditions, the symmetric solution  $\tilde{g}^{BB}$  in (24). To derive the equilibrium net profits (and contributions), we compute the firms' best response functions. Focusing first on lobbying in region a, to obtain the best response function of firm  $\beta$ , we need to solve problem BI defined in (A.5a)–(A.5b), in which firm  $\alpha$  lobbies in both regions while  $\beta$  no longer lobbies in region a. For any given  $\pi_{\alpha a}$  and  $\pi_{\alpha b}$ , the maximum amount of profits that firm  $\beta$  can make in region a ( $\pi_{\beta a}$ ) is constrained by the politician's participation constraint, which amounts to the following conditions:

$$V_a^{BB}\left(\tilde{g}_a^{BB}, \tilde{g}_b^{BB}, \pi_{\alpha a}, \pi_{\beta a}, \pi_{\alpha b}\right) = V_a^{BI}\left(\tilde{g}_a^{BI}, \tilde{g}_b^{BI}, \pi_{\alpha a}, \pi_{\alpha b}\right), \quad \text{if } \tilde{V}_a^{BI} \ge \tilde{V}_a^{OI}, \qquad (A.11a)$$

$$V_a^{BB}\left(\tilde{g}_a^{BB}, \tilde{g}_b^{BB}, \pi_{\alpha a}, \pi_{\beta a}, \pi_{\alpha b}\right) = V_a^{OI}\left(\tilde{g}_a^{OI}, \tilde{g}_b^{OI}, \pi_{\alpha b}\right), \quad \text{otherwise.}$$
(A.11b)

Given  $\pi_{\alpha a}$  and  $\pi_{\alpha b}$ , if policy maker a is better off under lobbying by firm  $\alpha$  than under no lobbying, i.e. if  $\tilde{V}_{a}^{BI} \geq \tilde{V}_{a}^{OI}$ , then Eq. (A.11a) says that firm  $\beta$  can increase its profits (and correspondingly reduce contributions) up to the point in which the policy maker is indifferent between being lobbed by both firms and being lobbed only by firm  $\alpha$ . On the contrary, if the policy maker is better off under no lobbying than under lobbying by firm  $\alpha$ , then (A.11b) says that firm  $\beta$  can increase its profits up to the point in which the policy maker is indifferent between being lobbed by both firms and not being lobbed, which is problem OI in (A.6a)–(A.6b). Routine algebra shows that  $\tilde{V}_{a}^{BI} \geq \tilde{V}_{a}^{OI}$  in (A.11a) can be written as  $\pi_{\alpha a} \leq \pi_7 \equiv \hat{\pi}_{\alpha a}^{NN} + mh^4$ , where  $\hat{\pi}_{\alpha a}^{NN} = h^2 \hat{g}^D$  is the amount of profits that a firm earns at home under decentralization and no lobbying (notice that  $\pi_7$  is independent of  $\pi_{\alpha b}$ ). Solving the equations in (A.11a) and (A.11b) for  $\pi_{\beta a}$ , we thus get, after some algebra, the best response function of firm  $\beta$  in region a:

$$\pi_{\beta a}^{*}(\pi_{\alpha a}) = \begin{cases} \pi_{5} & \text{if } \pi_{\alpha a} \leq \pi_{7}, \\ \pi_{6} + \frac{m}{1+m} (\hat{\pi}_{\alpha a}^{NN} - \pi_{\alpha a}) & \text{if } \pi_{7} < \pi_{\alpha a} \leq \pi_{9}, \\ \hat{\pi}_{\beta a}^{NN} & \text{if } \pi_{\alpha a} > \pi_{9}, \end{cases}$$
(A.12)

where  $\hat{\pi}_{\beta a}^{NN} = f^2 \hat{g}^D$  are the profits that a firm earns abroad under decentralization and no lobbying (the reservation utility of firm  $\beta$  in region *a*);  $\pi_5$ ,  $\pi_6$  and  $\pi_9$  are defined in Figure 2 in the text, in which the graph of  $\pi_{\beta a}^*(\pi_{\alpha a})$  is shown as the solid curve. Notice that the best response function is independent of  $\pi_{\alpha b}$ .

In a similar manner, to obtain the best response function of firm  $\alpha$  in region a, we need to solve problem OB defined in (A.6a)–(A.6b), in which firm  $\beta$  lobbies in both regions while  $\alpha$ no longer lobbies in its region. For any given  $\pi_{\beta a}$  and  $\pi_{\alpha b}$ , the maximum  $\pi_{\alpha a}$  that firm  $\alpha$  can make is constrained by the following conditions on politician's welfare in region a:

$$V_a^{BB}\left(\tilde{g}_a^{BB}, \tilde{g}_b^{BB}, \pi_{\alpha a}, \pi_{\beta a}, \pi_{\alpha b}\right) = V_a^{OB}\left(\tilde{g}_a^{OB}, \tilde{g}_b^{OB}, \pi_{\beta a}, \pi_{\alpha b}\right), \quad \text{if } \tilde{V}_a^{OB} \ge \tilde{V}_a^{OI}, \quad (A.13a)$$

$$V_a^{BB}\left(\tilde{g}_a^{BB}, \tilde{g}_b^{BB}, \pi_{\alpha a}, \pi_{\beta a}, \pi_{\alpha b}\right) = V_a^{OI}(\tilde{g}_a^{OI}, \tilde{g}_b^{OI}, \pi_{\alpha b}), \quad \text{otherwise.}$$
(A.13b)

The interpretation of these conditions is similar to the one given for conditions (A.11a)–(A.11b). Inequality  $\tilde{V}_a^{OB} \geq \tilde{V}_a^{OI}$  in (A.13a) can be written as  $\pi_{\beta a} \leq \pi_4 \equiv \hat{\pi}_{\beta a}^{NN} + (1+m)f^4$ . Solving

the equations in (A.13a) and (A.13b) for  $\pi_{\alpha a}$ , we finally get the best response function of firm  $\alpha$  in region a:

$$\pi_{\alpha a}^{*}(\pi_{\beta a}) = \begin{cases} \pi_{8} & \text{if } \pi_{\beta a} \leq \pi_{4}, \\ \pi_{9} + \frac{1+m}{m} (\hat{\pi}_{\beta a}^{NN} - \pi_{\beta a}) & \text{if } \pi_{4} < \pi_{\beta a} \leq \pi_{6}, \\ \hat{\pi}_{\alpha a}^{NN} & \text{if } \pi_{\beta a} > \pi_{6}, \end{cases}$$
(A.14)

where  $\pi_8$  is defined in Figure 2, in which the graph of  $\pi^*_{\alpha a}(\pi_{\beta \alpha})$  is shown as the dotted curve.

From (A.12) and (A.14), it is then immediate to see that lobbying by both firms in region a admits a set of Nash equilibria, defined by the profit pairs  $(\pi_{\alpha a}, \pi_{\beta a})$  such that

$$\pi_{\beta a} = \pi_6 + \frac{m}{1+m} (\hat{\pi}_{\alpha a}^{NN} - \pi_{\alpha a}), \quad \pi_7 \le \pi_{\alpha a} \le \pi_8.$$
(A.15)

By symmetry, the same set of Nash equilibria, this time in the profit pairs  $(\pi_{\beta b}, \pi_{\alpha b})$ , arises from lobbying by both firms in region *b*. Introducing a profit-distribution parameter  $\sigma^{ij} \in [0, 1]$ ,  $i, j \in \{B, I, O, N\}$ , the set of Nash equilibria (A.15) can thus be expressed as in (25) and (26) in the text, which are the equilibrium profits shown in Table A.1 for  $\sigma^{ij} = \sigma^{BB}$ . Finally, equilibrium contributions of the game *BB*, shown in Table A.2, are obtained from substitution of optimal public good supplies and profits into the contribution functions.

#### One firm lobbying the home region only and one lobbying both regions (IB)

Suppose that firm  $\alpha$  chooses I and firm  $\beta$  chooses B. Policy makers then solve the symmetric version of the problems defined in (A.5a)–(A.5b). Focusing first on region a, in which both firms lobby, for any given  $\pi_{\alpha a}$ , the best response function of firm  $\beta$  is defined by

$$V_{a}^{IB}\left(\tilde{g}_{a}^{IB}, \tilde{g}_{b}^{IB}, \pi_{\alpha a}, \pi_{\beta a}\right) = V_{a}^{II}(\tilde{g}_{a}^{II}, \tilde{g}_{b}^{II}, \pi_{\alpha a}), \quad \text{if } \tilde{V}_{a}^{II} \ge \tilde{V}_{a}^{NI}, \tag{A.16a}$$

$$V_a^{IB}\left(\tilde{g}_a^{IB}, \tilde{g}_b^{IB}, \pi_{\alpha a}, \pi_{\beta a}\right) = V_a^{NI}(\tilde{g}_a^{NI}, \tilde{g}_b^{NI}), \quad \text{otherwise}, \tag{A.16b}$$

where  $\tilde{V}_a^{NI}$  is defined in (A.8a) and  $\tilde{V}_a^{II}$  in (28a). Solving (A.16a)–(A.16b) for  $\pi_{\beta a}$  we get the best response function  $\pi^*_{\beta a}(\pi_{\alpha a})$  defined in (A.12). As for the best response function of firm  $\alpha$ , for any given  $\pi_{\beta a}$ , it is defined by:

$$V_a^{IB}\left(\tilde{g}_a^{IB}, \tilde{g}_b^{IB}, \pi_{\alpha a}, \pi_{\beta a}\right) = V_a^{NB}\left(\tilde{g}_a^{NB}, \tilde{g}_b^{NB}, \pi_{\beta a}\right), \quad \text{if } \tilde{V}_a^{NB} \ge \tilde{V}_a^{NI}, \tag{A.17a}$$

$$V_a^{IB}\left(\tilde{g}_a^{IB}, \tilde{g}_b^{IB}, \pi_{\alpha a}, \pi_{\beta a}\right) = V_a^{NI}(\tilde{g}_a^{NI}, \tilde{g}_b^{NI}), \quad \text{otherwise}, \tag{A.17b}$$

where  $\tilde{V}_a^{NB}$  is defined in (A.9a). Solving (A.17a)–(A.17b) for  $\pi_{\alpha a}$  we get the best response function  $\pi^*_{\alpha a}(\pi_{\beta a})$  defined in (A.14). Hence lobbying in region *a* under the strategy pair *IB* admits the set of Nash equilibria (25)–(26), for  $\pi_{\beta a}$ ,  $\pi_{\alpha a}$ ,  $\sigma^{ij} = \sigma^{IB}$  (see Table A.1).

As for lobbying in region b by firm  $\beta$ , the equilibrium profits  $\pi_{\beta b}$  (see Table A.1) are found by solving

$$V_b^{IB} \left( \tilde{g}_a^{IB}, \tilde{g}_b^{IB}, \pi_{\beta b}, \pi_{\beta a} \right) = V_b^{IO} (\tilde{g}_a^{IO}, \tilde{g}_b^{IO}, \pi_{\beta a}).$$
(A.18)

Finally, equilibrium contributions for the game IB (see Table A.2) are computed by substitutions of net profits and public good supplies into the compensating contribution schedules.

#### One firm lobbying abroad and the other lobbying both regions (OB)

Suppose that firm  $\alpha$  chooses O and firm  $\beta$  chooses B. Policy makers then solve (A.7a)–(A.7b). Focusing first on region b, in which both firms lobby, for any given  $\pi_{\beta b}$ , the best response function of firm  $\alpha$  is defined by

$$V_{b}^{OB}(\tilde{g}_{a}^{OB}, \tilde{g}_{b}^{OB}, \pi_{\beta a}, \pi_{\alpha b}, \pi_{\beta b}) = V_{b}^{NB}(\tilde{g}_{a}^{NB}, \tilde{g}_{b}^{NB}, \pi_{\beta b}), \quad \text{if } \tilde{V}_{b}^{NB} \ge \tilde{V}_{b}^{NO}, \tag{A.19a}$$

$$V_b^{OB}(\tilde{g}_a^{OB}, \tilde{g}_b^{OB}, \pi_{\beta a}, \pi_{\alpha b}, \pi_{\beta b}) = V_b^{NO}(\tilde{g}_a^{NO}, \tilde{g}_b^{NO}, \pi_{\beta a}), \quad \text{otherwise}, \tag{A.19b}$$

where  $\tilde{V}_b^{NB}$  and  $\tilde{V}_b^{NO}$  are defined in (A.9b) and (A.10b), respectively. Solving (A.19a)–(A.19b) for  $\pi_{\alpha b}$  we get the best response function  $\pi^*_{\alpha b}(\pi_{\beta b})$  defined in (A.12). As for the best response function of firm  $\beta$ , for any given  $\pi_{\alpha b}$ , it is defined by:

$$V_{b}^{OB}(\tilde{g}_{a}^{OB}, \tilde{g}_{b}^{OB}, \pi_{\beta a}, \pi_{\alpha b}, \pi_{\beta b}) = V_{b}^{OO}(\tilde{g}_{a}^{NB}, \tilde{g}_{b}^{NB}, \pi_{\beta a}, \pi_{\alpha b}), \quad \text{if } \tilde{V}_{b}^{OO} \ge \tilde{V}_{b}^{NO}, \quad (A.20a)$$

$$V_b^{OB}(\tilde{g}_a^{OB}, \tilde{g}_b^{OB}, \pi_{\beta a}, \pi_{\alpha b}, \pi_{\beta b}) = V_b^{NO}(\tilde{g}_a^{NO}, \tilde{g}_b^{NO}, \pi_{\beta a}), \quad \text{otherwise},$$
(A.20b)

where  $\tilde{V}_a^{OO}$  is defined in (32a). Solving (A.20a)–(A.20b) for  $\pi_{\beta b}$  we get the best response function  $\pi^*_{\beta b}(\pi_{\alpha a})$  defined in (A.14). Hence lobbying in region *b* under *OB* admits the set of Nash equilibria (25) and (26), for  $\pi_{\alpha b}$ ,  $\pi_{\beta b}$ ,  $\sigma^{ij} = \sigma^{OB}$  (see Table A.1).

As for lobbying in region a by firm  $\beta$ , the equilibrium profits  $\pi_{\beta a}$  (see Table A.1) are found by solving

$$V_{a}^{OB}\left(\tilde{g}_{a}^{OB}, \tilde{g}_{b}^{OB}, \pi_{\alpha b}, \pi_{\beta a}\right) = V_{a}^{OI}(\tilde{g}_{a}^{OI}, \tilde{g}_{b}^{OI}, \pi_{\alpha b}).$$
(A.21)

Finally, equilibrium contributions for the game OB (see Table A.2) are computed by substitutions of net profits and public good supplies into the compensating contribution schedules.

#### One firm not lobbying and the other lobbying both regions (NB)

Suppose that firm  $\alpha$  chooses N and  $\beta$  chooses B. Policy makers solve (A.9a)–(A.9b). Assume now that firm  $\beta$  is not lobbying. Policy makers maximize  $V_a^{NN} = \mu W_a$  and  $V_b^{NN} = \mu W_b$ . The solution is the no-lobbying optimal public good supply  $\hat{g}^D$  for both  $g_a$  and  $g_b$ . Solving

$$\begin{split} V_a^{NB} \left( \tilde{g}_a^{NB}, \tilde{g}_b^{NB}, \pi_{\beta a} \right) &= V_a^{NI} \left( \hat{g}_a^{NI}, \hat{g}_b^{NI} \right), \\ V_b^{NB} \left( \tilde{g}_a^{NB}, \tilde{g}_b^{NB}, \pi_{\beta a}, \pi_{\beta b} \right) &= V_b^{NO} \left( \hat{g}_a^{NO}, \hat{g}_b^{NO}, \pi_{\beta a} \right), \end{split}$$

for  $\pi_{\beta a}$  and  $\pi_{\beta b}$  we get the equilibrium profits that a firm lobbying both regions makes at home and abroad when the other firm is not lobbying (see Table A.1). Profits at home and abroad of the no-lobbying firm  $\alpha$  are computed by substituting the optimal public good supplies into the corresponding profit functions. Finally, equilibrium contributions for the game NB (see Table A.2) are computed by simple substitutions of net profits and public good supplies into the compensating contribution schedules of firm  $\beta$ .

#### Both firms lobbying only the home region (II)

When both firms lobby only the home region, policy makers' solve problems (28a)–(28b) in the text. From the corresponding first order conditions, we get the optimal public good supplies

 $\tilde{g}^{II}$  in (30). To compute the equilibrium profits, assume that  $\beta$  lobbies at home (I), while  $\alpha$  does not lobby (N). Policy makers then solve (A.8a)–(A.8b). Solving the equation

$$V_a^{II}(\tilde{g}_a^{II}, \tilde{g}_b^{II}, \pi_{\alpha a}) = V_a^{NI}(\tilde{g}_a^{NI}, \tilde{g}_b^{NI})$$

for the home profits  $\pi_{\alpha a}$ , and then adding the "abroad" profits,  $f^2 \tilde{g}^{II}$ , we get that total profits  $\tilde{\pi}^{II}$  in (31). Equilibrium contributions, shown in Table A.2, are obtained by substituting (home) net profits and public good supply into firm  $\alpha$ 's compensating contribution schedule to region a.

## One firm no lobbying and the other lobbying the home region (NI)

Suppose that firm  $\beta$  chooses I and firm  $\alpha$  chooses N. Policy makers solve (A.8a)–(A.8b). Assuming that firm  $\beta$  is not lobbying, policy makers maximize  $V_a^{NN} = \mu W_a$  and  $V_b^{NN} = \mu W_b$ . The solution is the no-lobbying optimal public good supply  $\hat{g}^D$  for both  $g_a$  and  $g_b$ . Solving

$$V_b^{NI}\left(\tilde{g}_a^{NI}, \tilde{g}_b^{NI}, \pi_{\beta b}\right) = V_b^{NN}\left(\hat{g}_a^D, \hat{g}_b^D\right),$$

for  $\pi_{\beta b}$  we get the equilibrium profits that a firm makes at home when lobbying only at home while the other firm is not lobbying (see Table A.1). Firm  $\beta$ 's profits abroad, and profits at home and abroad of the no-lobbying firm  $\alpha$  are computed by substituting the optimal public good supplies into the corresponding profit functions. Equilibrium contributions for the game NI (see Table A.2) are computed by substituting net profits and public good supplies into firm  $\beta$ 's compensating contribution schedule.

## One firm lobbying abroad and the other lobbying the home region (OI)

Suppose that firm  $\alpha$  chooses O and firm  $\beta$  chooses I. Policy makers then solve (A.6a)–(A.6b). In region b, in which both firms lobby, for any given  $\pi_{\beta b}$ , the best response function of firm  $\alpha$  is defined by

$$V_{b}^{OI}(\tilde{g}_{a}^{OI}, \tilde{g}_{b}^{OI}, \pi_{\alpha b}, \pi_{\beta b}) = V_{b}^{NI}(\tilde{g}_{a}^{NI}, \tilde{g}_{b}^{NI}, \pi_{\beta b}), \quad \text{if } \tilde{V}_{b}^{NI} \ge \tilde{V}_{b}^{NN}, \tag{A.22a}$$

$$V_b^{OI}(\tilde{g}_a^{OI}, \tilde{g}_b^{OI}, \pi_{\alpha b}, \pi_{\beta b}) = V_b^{NN}(\tilde{g}_a^{NN}, \tilde{g}_b^{NN}), \quad \text{otherwise},$$
(A.22b)

where  $\tilde{V}_b^{NI}$  is defined in (A.8b). Solving (A.22a)–(A.22b) for  $\pi_{\alpha b}$  we get the best response function  $\pi^*_{\alpha b}(\pi_{\beta b})$  defined in (A.12). As for the best response function of firm  $\beta$ , for any given  $\pi_{\alpha b}$ , it is defined by:

$$V_{b}^{OI}(\tilde{g}_{a}^{OI}, \tilde{g}_{b}^{OI}, \pi_{\alpha b}, \pi_{\beta b}) = V_{b}^{ON}(\tilde{g}_{a}^{ON}, \tilde{g}_{b}^{ON}, \pi_{\alpha b}), \quad \text{if } \tilde{V}_{b}^{ON} \ge \tilde{V}_{b}^{NN}, \tag{A.23a}$$

$$V_b^{OI}(\tilde{g}_a^{OI}, \tilde{g}_b^{OI}, \pi_{\alpha b}, \pi_{\beta b}) = V_b^{NN}(\tilde{g}_a^{NO}, \tilde{g}_b^{NO}), \quad \text{otherwise.}$$
(A.23b)

Solving (A.23a)–(A.23b) for  $\pi_{\beta b}$  we get the best response function  $\pi^*_{\beta b}(\pi_{\alpha a})$  defined in (A.14). Hence lobbying in region *b* under *OI* admits the set of Nash equilibria (25) and (26) in terms of firms' net profits  $\pi_{\alpha b}$ ,  $\pi_{\beta b}$ , for  $\sigma^{ij} = \sigma^{OI}$  (see Table A.1).

Finally, equilibrium contributions for the game OI (see Table A.2) are computed by substituting net profits and public good supplies into the compensating contribution schedules.

#### Both firms lobbying only abroad (OO)

When both firms lobby only abroad, the policy makers' solve problems (32a)–(32b) in Section 4.2. From the corresponding first order conditions, we get the optimal public good supplies  $\tilde{g}^{OO}$  in (34). Assume now that firm  $\alpha$  does not lobby while firm  $\beta$  lobbies abroad. The game is NO, and policy makers solve (A.10a)–(A.10b). Solving the equation

$$V_b^{OO}\left(\tilde{g}_a^{OO}, \tilde{g}_b^{OO}, \pi_{\alpha b}, \pi_{\beta a}\right) = V_b^{NO}\left(\tilde{g}_a^{NO}, \tilde{g}_b^{NO}, \pi_{\beta a}\right)$$

for  $\pi_{\alpha b}$  we obtain the equilibrium profits that a firm makes abroad when both firms are lobbying abroad only (see Table A.1). Adding the home profits, we get total profits  $\tilde{\pi}^{OO}$  shown in (35). Equilibrium contributions, shown in Table A.2, are computed by substituting (abroad) net profits and public good supply into firm  $\alpha$ 's compensating contribution schedule to region b.

#### One firm no lobbying and the other lobbying abroad (NO)

Suppose that firm  $\beta$  chooses O and firm  $\alpha$  chooses N. Policy makers solve (A.10a)–(A.10b). Assuming that firm  $\beta$  is not lobbying, policy makers maximize  $V_a^{NN} = \mu W_a$  and  $V_b^{NN} = \mu W_b$ . The solution is the no-lobbying optimal public good supply  $\hat{g}^D$  for both  $g_a$  and  $g_b$ . Solving

$$V_{a}^{NO}\left(\tilde{g}_{a}^{NO},\tilde{g}_{b}^{NO},\pi_{\beta a}\right)=V_{a}^{NN}\left(\hat{g}_{a}^{D},\hat{g}_{b}^{D}\right)$$

for  $\pi_{\beta a}$  we obtain the equilibrium profits that a firm earns abroad when lobbying only abroad while the other firm is not lobbying (see Table A.1). Firm  $\beta$ 's profits at home, and profits at home and abroad of the no-lobbying firm  $\alpha$  are computed by substituting the optimal public good supplies into the corresponding profit functions. Equilibrium contributions for the game NO (see Table A.2) are computed by substituting net profits and public good supply into firm  $\beta$ 's contribution schedule.

## A.3 Proof of Proposition 2

From the where-to-lobby game in Table 1,  $\forall \mu \in (0, 1]$  and  $\forall \delta \in [1, \delta_{\max}]$ , plain algebra shows that II is a Nash equilibrium of the where-to-lobby game if and only if  $2mh^2f^2(1-\sigma) - (1+m)f^4 \ge 0$ , i.e.  $\mu \le \mu_I(\delta; c, \sigma) \equiv \max\left\{1 - \frac{f^2}{2(1-\sigma)h^2}, 0\right\}$ , for  $\sigma \ne 1$ . Similarly, OO is a Nash equilibrium if and only if  $mh^4 - 2(1+m)h^2f^2\sigma \le 0$ , i.e.  $\mu \ge \mu_O(\delta; c, \sigma) = \max\left\{1 - \frac{2f^2}{h^2}\sigma, 0\right\}$ . Finally, BB is a Nash equilibrium if and only if  $2mh^2f^2(1-\sigma) - (1+m)f^4 < 0$  and  $mh^4 - 2(1+m)h^2f^2\sigma < 0$ ; hence  $\mu > \mu_I(\delta; c, \sigma)$  and  $\mu < \mu_O(\delta; c, \sigma)$ . Both  $\mu_I(\delta; c, \sigma)$  and  $\mu_O(\delta; c, \sigma)$  are monotonically increasing in  $\delta$  and  $\mu_O(\delta; c, \sigma) - \mu_I(\delta; c, \sigma) \ge 0$  for all admissible values of the parameters, since  $1 - \frac{2f^2}{h^2}\sigma - \left(1 - \frac{f^2}{2(1-\sigma)h^2}\right)$  implies  $4\sigma^2 - 4\sigma + 1 \ge 0$  for  $\sigma \in [0, 1]$ .

## A.4 Proof of Proposition 3

From Table 1 it is  $\Delta \pi_{\rho}^{BB} - \Delta \pi_{\rho}^{II} = (1+m)f^2 + 2h^2f^2(1-\sigma) \ge 0 \ \forall \mu \in (0,1], \ \forall \delta \in [1, \delta_{\max}]$ and  $\forall \sigma \in [0,1]$ . Hence, *BB* always Pareto dominates *II* in terms of firms' net profits. Again from Table 1 it is  $\Delta \pi^{BB} - \Delta \pi^{OO} = mh^4 - 2h^2f^2\sigma$ , and  $\Delta \pi^{BB} - \Delta \pi^{OO} \ge 0$  if and only if  $\mu \leq \mu_{BO}(\delta; c, \sigma) \equiv \frac{h^2}{h^2 + 2f^2\sigma}$ . Plain algebra shows that  $\mu_{BO}(\delta; c, \sigma) \geq \mu_O(\delta; c, \sigma) \ \forall \sigma \in [0, 1]$  and  $\forall \delta \in [1, \delta_{\max}]$ , since  $4f^4\sigma^2 \geq 0$ . Thus, when the Nash equilibrium is OO, it is Pareto efficient in terms of firms' net profits if and only if  $\mu \geq \mu_{BO}(\delta; c, \sigma)$ , so that  $\Delta \pi_{\rho}^{BB} \leq \Delta \pi_{\rho}^{OO}$ ; otherwise it is dominated by the strategy pair BB.

## A.5 Proof of Proposition 5

As for the comparison of net profits, using (20), (27), (31) and (35) it is  $\tilde{\pi}^C - \tilde{\pi}_{\rho}^{BB} = f^4 + 2h^2 f^2 \sigma^{BB} > 0$ ,  $\tilde{\pi}^C - \tilde{\pi}_{\rho}^{II} = 2h^2 f^2 + (2+m)f^4 \ge 0$  and  $\tilde{\pi}^C - \tilde{\pi}_{\rho}^{OO} = mh^4 + f^4 > 0$ , which shows that profits are higher under centralization. As for contributions, from (21) and Table A.2, it is  $\tilde{s}_{\alpha a}^{BB} + \tilde{s}_{\alpha b}^{BB} - \tilde{s}^C = f^4 + 2h^2 f^2 \sigma^{BB} > 0$ ,  $\tilde{s}^C - \tilde{s}_{\alpha a}^{II} = mf^4 + 2mh^2 f^2 \ge 0$  and  $\tilde{s}^C - \tilde{s}_{\alpha b}^{OO} = mh^4 - f^4 + 2mh^2 f^2$ . From the latter one obtains that  $\tilde{s}^C \ge \tilde{s}_{\alpha b}^{OO}$  if and only if  $\mu \le \mu^S(\delta; c) \equiv \frac{h^2(h^2+2f^2)}{(h^2+f^2)^2}$ .  $\mu^S(\delta; c)$  is monotonically increasing in  $\delta$ , with  $\mu^S(1; c) = \frac{3}{4}$ ,  $\mu^S(\delta_{\max}; c) = 1$ . As, for values of  $\delta \ne \delta_{\max}$ , there are values of  $\sigma$  such that  $\mu^S(\delta; c) > \mu_O(\delta; c, \sigma)$ , meaning that the region in which OO is a Nash equilibrium is divided into two areas:  $\tilde{s}^C < \tilde{s}_{\alpha b}^{OO}$ , for  $\mu > \mu^S$ ;  $\tilde{s}^C \ge \tilde{s}_{\alpha b}^{OO}$ , otherwise. As for public good provision and social welfare, from (18) and (24) it is  $\hat{g}^C = \tilde{g}^{BB}$ , which implies that aggregate social welfare is the same under centralization and under the equilibria BB. By the comparison of (18) and (30) it follows that  $\tilde{g}^{II} \le \hat{g}^C$ ; aggregate social welfare is larger in the decentralized equilibrium II since  $|\tilde{g}^{II} - \hat{g}^C| \le |\tilde{g}^C - \hat{g}^C|$ , given that  $\hat{g}^C$  maximizes social welfare, which is quadratic in public goods supply. Finally, using (18) and (34) one can see that  $|\tilde{g}^{OO} - \hat{g}^C| \le |\tilde{g}^C - \hat{g}^C|$ ; aggregate social welfare is larger in the decentralized equilibrium IO.

# **B** Appendix: Lobbying for the market

## B.1 Proof of Lemma 1

We first derive the optimal public goods levels by maximizing  $W^{J_aK_b} = W_a^{J_aK_b} + W_b^{J_aK_b}$ ,  $J, K = \{H, F\}$ , as defined in (40)–(43), with respect to  $g_a$  and  $g_b$ . This gives

$$\hat{g}_{a}^{H_{a}H_{b}} = \hat{g}_{b}^{H_{a}H_{b}} = \hat{g}_{a}^{H_{a}F_{b}} = \hat{g}_{b}^{F_{a}H_{b}} = 3H^{2}, \ \hat{g}_{a}^{F_{a}F_{b}} = \hat{g}_{b}^{F_{a}F_{b}} = \hat{g}_{b}^{H_{a}F_{b}} = \hat{g}_{a}^{F_{a}H_{b}} = 3F^{2}.$$

Monopoly profits when supplying the home and the foreign region are  $3H^4$  and  $3F^4$ , respectively. Thus, given  $S_{\rho}^H$  and  $S_{\rho}^F$ , with  $0 \leq S_{\rho}^H \leq 3H^4$  and  $0 \leq S_{\rho}^F \leq 3F^4$ , the politician's value functions in the four possible cases are

$$\begin{split} V^{H_aH_b} &= \mu \frac{9H^4}{2} + (1-\mu)(S^H_\alpha + S^H_\beta) + 2\mu\bar{y}, \\ V^{H_aF_b} &= \mu \frac{9(H^4 + F^4)}{4} + (1-\mu)(S^H_\alpha + S^F_\alpha) + 2\mu\bar{y}, \\ V^{F_aH_b} &= \mu \frac{9(H^4 + F^4)}{4} + (1-\mu)(S^H_\beta + S^F_\beta) + 2\mu\bar{y}, \\ V^{F_aF_b} &= \mu \frac{9F^4}{2} + (1-\mu)(S^F_\alpha + S^F_\beta) + 2\mu\bar{y}. \end{split}$$

Consider firm  $\alpha$  (the same argument holds true for firm  $\beta$ ). Given  $S^H_{\beta}$  and  $S^F_{\beta}$  the government chooses  $H_aH_b$  if and only if  $V^{H_aH_b} \geq V^{H_aF_b}$ ,  $V^{H_aH_b} \geq V^{F_aH_b}$ ,  $V^{H_aH_b} \geq V^{F_aF_b}$ ; after some algebra these inequalities reduce to  $S^H_{\alpha} \geq T^H(S^F_{\beta})$  and  $S^F_{\alpha} \leq T^F(S^H_{\beta})$ , where

$$\begin{split} T^{H}(S^{F}_{\beta}) &= \max\left\{-\frac{9\mu(H^{4}-F^{4})}{4(1-\mu)} + S^{F}_{\beta}, \ 0\right\},\\ T^{F}(S^{H}_{\beta}) &= \min\left\{\frac{9\mu(H^{4}-F^{4})}{4(1-\mu)} + S^{H}_{\beta}, \ 3F^{4}\right\}. \end{split}$$

Analogously one gets that the government chooses  $F_aF_b$  if and only if  $S^H_{\alpha} \leq T^H(S^F_{\beta})$  and  $S^F_{\alpha} \geq T^F(S^H_{\beta})$ ,  $H_aF_b$  if and only if  $S^H_{\alpha} > T^H(S^F_{\beta})$  and  $S^F_{\alpha} > T^F(S^H_{\beta})$ , and  $F_aH_b$  if and only if  $S^H_{\alpha} < T^H(S^F_{\beta})$  and  $S^F_{\alpha} < T^F(S^H_{\beta})$ . The profit function of the firm is then defined as

$$\Pi_{\alpha}(S^{H}_{\alpha}, S^{F}_{\alpha}; S^{H}_{\beta}, S^{F}_{\beta}) = \begin{cases} 3H^{4} - S^{H}_{\alpha} & \text{if } S^{H}_{\alpha} \geq T^{H}(S^{F}_{\beta}) \text{ and } S^{F}_{\alpha} \leq T^{F}(S^{H}_{\beta}), \\ 3F^{4} - S^{F}_{\alpha} & \text{if } S^{H}_{\alpha} \leq T^{H}(S^{F}_{\beta}) \text{ and } S^{F}_{\alpha} \geq T^{F}(S^{H}_{\beta}), \\ 3(H^{4} + F^{4}) - S^{H}_{\alpha} - S^{F}_{\alpha} & \text{if } S^{H}_{\alpha} > T^{H}(S^{F}_{\beta}) \text{ and } S^{F}_{\alpha} > T^{F}(S^{H}_{\beta}), \\ 0 & \text{if } S^{H}_{\alpha} < T^{H}(S^{F}_{\beta}) \text{ and } S^{F}_{\alpha} < T^{F}(S^{H}_{\beta}). \end{cases}$$

Profit maximization requires the firm to set  $S^H_{\alpha} = T^H(S^F_{\beta}) + \varepsilon$  and  $S^F_{\alpha} = T^F(S^H_{\beta}) + \varepsilon$ , with  $\varepsilon > 0$  as close as possible to zero. Since the same profit maximizing behavior holds true for firm  $\beta$ , the two firms will engage in a Bertrand-type competition in contributions, leading to the unique Nash equilibrium (pure) strategy profile:  $\hat{S}^F_{\rho} = 3F^4$  and  $\hat{S}^H_{\rho} = \max\left\{\hat{T}^H, 0\right\}$ , with  $\hat{T}^H = T^H(\hat{S}^F_{\rho})$  as defined in (45). The corresponding politician's value function (46) follows immediately by substituting  $\hat{S}^H_{\rho}$  into the expression for  $V^{H_a H_b}$  above.

## **B.2** Proof of Proposition 6

From  $\hat{T}^H = 0$ , with  $\hat{T}^H$  defined in (45), one gets

$$\mu_2(\delta;c) = \frac{4F^4}{F^4 + 3H^4},\tag{B.1}$$

where H and F are defined in (39). Eq. (B.1) divides the closed set  $S = (\mu, \delta) \in [0, 1] \times [1, \delta_{\max}]$ in two regions (see Figure B.1):  $\hat{S}^H > 0$  for  $\mu < \mu_2$ , and  $\hat{S}^H = 0$  otherwise.  $\mu_2(\delta; c) \in C^2$  is monotonically decreasing in  $\delta$ , with  $\mu_2(1; c) = 1$  and  $\mu_2(\delta_{\max}; c) = \frac{4}{49} = .082$ . From

$$\hat{V}^{hf} - \mu \frac{9H^4}{2} - 2(1-\mu)\hat{T}^H - 2\mu\bar{y} = 0$$

it is

$$\mu_1(\delta;c) = \frac{12F^4}{3F^4 + [(h+f)^2 + 2(h^2 + f^2)]^2},\tag{B.2}$$

where h and f are defined in (5). One can see that  $\mu_1(\delta;c) \in C^2$ ,  $\mu_1(1;c) = \frac{972}{1267} \approx .767$ ,  $\mu_1(\delta_{\max};c) = \frac{4}{49}$ , and that  $\mu_1(\delta;c)$  and  $\mu_2(\delta;c)$  have a unique intersection at  $\delta = \delta_1(c) \equiv \frac{5+17c}{22c}$  for  $\delta \in [1, \delta_{\max})$ , for which  $\mu = \frac{334084}{786289} \approx .425$ . Thus, for  $\delta \in [1, \delta_1]$ , the locus defined by Eq.



Figure B.1: An illustration of Proposition 6

(B.2) separates the subset of S in which  $\hat{S}^H > 0$  into two subsets such that:  $\hat{V}^{hf} > \hat{V}^H$  for  $\mu > \mu_1$  and  $\hat{V}^{hf} \le \hat{V}^H$  otherwise, proving the first part of the proposition. For  $\delta \in (\delta_1, \delta_{\max})$ , if  $\mu \le \mu_1$  then  $\hat{V}^{hf} < \hat{V}^H$  since  $\mu_1 > \mu_2$ . If  $\mu \ge \mu_2$  then  $\hat{S}^H = 0$ , while if  $\mu < \mu_2$  then  $\hat{S}^H > 0$ . Define  $\Psi(\mu, \delta; c) = \hat{V}^{hf} - \mu \frac{9H^4}{4}, \Psi \in C^2$ . Since  $\Psi(\mu, 1; c) > 0, \Psi(\mu, \delta_{\max}; c) = 0$  and there is a unique root at  $\delta = \delta_1$  for  $\delta \in [1, \delta_{\max})$ , then  $\Psi < 0$  for all  $\delta \in (\delta_1, \delta_{\max})$ , proving that only the home firm enters the market without paying any contribution.

## B.3 Proof of Lemma 2

The proof is conducted in three steps.

Step 1. Both regional governments admit one firm only. By deriving the optimal public goods levels through the maximization in  $g_a$  and  $g_b$ , respectively, of  $W_a^{J_aK_b}$  and  $W_b^{J_aK_b}$ ,  $J, K = \{H, F\}$ , as defined in (40)–(43), and given  $S_{\rho}^{H}$  and  $S_{\rho}^{F}$ ,  $\rho = \{\alpha, \beta\}$ , with  $0 \leq S_{\rho}^{H} \leq 3H^4$  and  $0 \leq S_{\rho}^{F} \leq 3F^4$ , we obtain region *a* politician's value functions in the four possible cases

$$\begin{split} V_a^{H_aH_b} &= \mu \frac{9H^4}{4} + (1-\mu)S_{\alpha}^H + \mu \bar{y}, \\ V_a^{H_aF_b} &= \mu \left(\frac{9H^4}{4} + F^4\right) - \mu S_{\alpha}^F + (1-\mu)S_{\alpha}^H + \mu \bar{y}, \\ V_a^{F_aH_b} &= \mu \frac{F^4}{4} + S_{\beta}^F + \mu \bar{y}, \\ V_a^{F_aF_b} &= \mu \frac{5F^4}{4} - \mu S_{\alpha}^F + S_{\beta}^F + \mu \bar{y}. \end{split}$$

Given  $S_{\beta}^{F}$ , it is a (weakly) dominant strategy for region a (and symmetrically the same holds true for region b) to choose the home firm if and only if  $V_{a}^{H_{a}H_{b}} \geq V_{a}^{F_{a}H_{b}}$  and  $V_{a}^{H_{a}F_{b}} \geq V_{a}^{F_{a}F_{b}}$ . These two inequalities are satisfied for the same condition, i.e.

$$S^{H}_{\alpha}(S^{F}_{\beta}) \ge \max\left\{-\frac{\mu(9H^{4}-F^{4})}{4(1-\mu)} + \frac{S^{F}_{\beta}}{1-\mu}, 0\right\}.$$

Bertrand competition in contributions implies that  $\hat{S}^F_{\rho} = F^4$  and thus it is

$$\hat{S}_{\rho}^{H} = \max\left\{-\frac{\mu(9H^{4} - F^{4})}{4(1-\mu)} + \frac{F^{4}}{1-\mu}, 0\right\}$$

proving Equation (47) in the lemma. One needs to check that  $\hat{S}^{H}_{\rho} \leq 3H^{4}$ . For  $\mu \neq 1$ , this requires  $\mu(\delta) \leq \mu^{T}(\delta) \equiv 4\frac{3H^{4}-F^{4}}{3H^{4}+F^{4}}$ . By recalling (39), it is immediate to show that it is  $\mu^{T}(1) = 2$  and  $\frac{\partial \mu^{T}(\delta)}{\partial \delta} > 0$ . Hence  $\hat{S}^{H}_{\rho}$  is always smaller than the profits realized in the home region.

Thus, when one firm only is allowed to enter a regional market, the home firm wins the contest for the market and the politician's value function (in each region) is  $\hat{V}_a^{H_aH_b}$  in Table 2.

Step 2. Both regional governments allow both firms in their domestic markets. This case has been examined in Section 3, where policy without lobbying has been described. Using the optimal public good provision given in (13) and substituting it into (10), region *a* politician's value function when both firms are allowed to enter their market is  $\hat{V}_a^{hf_ahf_b}$ , shown in Table 2. Step 3. One regional government admits one firm only and the other one admits both. Suppose, without loss of generality, that region *a* lets both firms in, while region *b* allows only one of them to enter its regional market. If firm  $\beta$  gets region *b*'s market, social welfare becomes

$$\begin{split} W_a^{hf_aH_b} &= \mu \frac{(h+f)^2 g_a + 2h^2 g_a}{2} - \frac{g_a^2}{4}, \\ W_b^{hf_aH_b} &= \mu \frac{3H^2 g_b + 2f^2 g_a}{2} - \frac{g_b^2}{4} + (1-\mu)S_\beta^H \end{split}$$

On the other hand, in the case in which firm  $\alpha$  gets region b's market, the corresponding social welfare functions are

$$\begin{split} W_a^{hf_aF_b} &= \mu \frac{(h+f)^2 g_a + 2h^2 g_a + 2F^2 g_b - 2S_{\alpha}^F}{2} - \frac{g_a^2}{4}, \\ W_b^{hf_aF_b} &= \mu \frac{F^2 g_b + 2f^2 g_a + 2S_{\alpha}^F}{2} - \frac{g_b^2}{4} + (1-\mu)S_{\alpha}^F. \end{split}$$

By maximizing each regional social welfare function in the local public good supply, one obtains the corresponding politicians' value functions

$$\begin{split} \hat{V}_{a}^{hf_{a}H_{b}} &= \mu \frac{[(h+f)^{2}+2h^{2}]^{2}}{4}, \\ V_{b}^{hf_{a}H_{b}} &= \mu \frac{9H^{4}+4f^{2}[(h+f)^{2}+2h^{2}]}{4} + (1-\mu)S_{\beta}^{H}, \\ V_{a}^{hf_{a}F_{b}} &= \mu \frac{[(h+f)^{2}+2h^{2}]^{2}+4F^{4}}{4} - \mu S_{\alpha}^{F}, \\ V_{b}^{hf_{a}F_{b}} &= \mu \frac{F^{4}+4f^{2}[(h+f)^{2}+2h^{2}]}{4} + S_{\alpha}^{F}. \end{split}$$

Region b allows firm  $\beta$  in if and only if  $V_b^{hf_aH_b} \geq V_b^{hf_aF_b}$  that requires

$$S_{\beta}^{H}(S_{\alpha}^{F}) \ge \max\left\{-\frac{\mu(9H^{4}-F^{4})}{4(1-\mu)} + \frac{S_{\alpha}^{F}}{1-\mu}, 0\right\}.$$

By Bertrand competition,  $\hat{S}^F_{\alpha} = F^4$  and

$$\hat{S}^{H}_{\beta} = \max\left\{-\frac{\mu(9H^{4}-F^{4})}{4(1-\mu)} + \frac{F^{4}}{1-\mu}, 0\right\},\$$

where  $\hat{S}^{H}_{\beta} > 0$  for  $\mu < \frac{4F^4}{9H^4 - F^4}$ . Moreover, by the same argument in Step 2,  $\hat{S}^{H}_{\beta} \leq 3H^4$ . Thus, substituting  $\hat{S}^{H}_{\beta}$  into  $V^{hf_aH_b}_{b}$  the region *b* politician's value function is  $\hat{V}^{hf_aH_b}_{b}$  in Table 2. The same applies symmetrically when region *b* let both firms in, while region *a* allows only one of them to enter its regional market.

## B.4 Proof of Proposition 8

Considering the game in Table 2, it is a (weakly) dominant strategy for both regions to admit one firm only if and only if  $\hat{V}_{a}^{hf_{a}H_{b}} \geq \hat{V}_{a}^{hf}$  and  $\hat{V}_{a}^{H_{a}H_{b}} \geq \hat{V}_{a}^{hf_{a}H_{b}}$ . These inequalities imply (i)  $\mu \leq \mu_{6}(\delta;c) \equiv \frac{4F^{4}}{[(h+f)^{2}+2h^{2}]^{2}}$  for  $\mu < \mu_{5}(\delta;c)$ , where  $\mu_{5}(\delta;c)$  is defined in (48), and (ii)  $\mu\{9H^{4} - [(h+f)^{2}+2h^{2}]^{2}\} \geq 0$  for  $\mu \geq \mu_{5}(\delta;c)$ . Condition (ii) is always satisfied for all  $\delta \in [1, \delta_{\max}]$  and  $c \in (0, 1)$ ; hence one-firm in each region is the unique Nash equilibrium for  $\mu \geq \mu_{5}(\delta;c)$ . As for condition (i), it is always satisfied for all  $\delta \in [1, \delta_{\max}]$  and  $c \in (0, 1)$ , since  $\mu_{6}(\delta;c) \geq \mu_{5}(\delta;c)$ . The latter inequality follows by a continuity argument from  $\mu_{6}(1;c) = \frac{36}{55} > \mu_{5}(1;c) = \frac{1}{2}, \mu_{6}(\delta_{\max};c) = \mu_{5}(\delta_{\max};c) = \frac{4}{143}, \text{ and } \mu_{6}(\delta;c) \neq \mu_{5}(\delta;c)$  for all  $\delta \in [1, \delta_{\max})$ . Hence one-firm in each region is the unique Nash equilibrium also for  $\mu < \mu_{5}(\delta;c)$ . In both cases, by Lemma 2, it is the home firm to gain access to the market.

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