Horizontal equity in a regional context

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1 The horizontal equity principle in a regional context

In this paper, we formulate a composite principle of fiscal equity and we study the consistency of the current public expenditure in Italy with such principle.

The starting point is the idea that there are some spheres of public intervention with respect to which any differential fiscal treatment of individuals across jurisdictions is inequitable; whereas, in some other areas of public intervention, fiscal differentiation across regions can be legitimate.

This principle of fiscal equity can be justified on the basis of the horizontal equity principle.

Definition 1 *The horizontal equity principle requires the equal treatment of equals.*

This requirement can not be applied in any meaningful way until the concepts of "equals" and "equal treatment" have been precisely defined. The exact definition of equals and equal treatment depends upon the particular conception of equity one decides to adopt. Suppose, for the moment, that equal treatment refers to the amount of public goods and services provided by the government (assuming away any difficulty in measuring this supply). What remains to be defined is the concept of equals. Equals could be those with equal income, those with equal utility, those with the same opportunities for choice, and so on. Suppose we can exhaustively describe two individuals A and B by a list of (observable) personal characteristics, respectively a_i and b_i, each of them partitioned in two groups, with $1 \leq i \leq m+n$:

$A = (a_1, ..., a_m; a_{m+1}, ..., a_{m+n})$

$B = (b_1, ..., b_m; b_{m+1}, ..., b_{m+n})$

Suppose now we stipulate that two individuals are deemed normatively "equals" if and only if they are equals in the first m attributes (e.g. they have the same income, marital status, family composition, occupation, and so on). This amount to say that the first m attributes are relevant for different treatment, but the remaining are not. Hence the Horizontal Equity Principle could be restated as requiring the equal treatment of individuals with the same relevant characteristics.

In other words, defining the equals implies defining what constitutes a *legitimate basis* for discrimination and what does not. We need an ethical principle in order to answer the following question: what are these relevant characteristics? What does constitute a legitimate basis for distinction and what does not? Presumably, the answer will depend on the political issues we are concerned about: the income level of individual will be relevant for the definition of his personal income tax burden; the same information could be considered as irrelevant in the definition of an health care policy and so on. In any practical application of the notion, the degree of observability and the reliance of these observations on the relevant issues will also be a matter of importance.

In the current context, we avoid all these issues and focus instead on the following question: is the *location of the individual* (i.e. the region or city of domicile) a relevant attribute for different treatment? We argue that the answer depends on the particular issue at hand: for some issues (those where the Constitution of a country or its current practise allows for autonomous decisions at local level) differential treatment of individual based on domicile is legitimate; for some other issues, (those whose responsibility is assigned to the central level of government) it will not be the case. To be precise, suppose an individual h is completely described by a set of traits $\mathbf{c} := \left(c_h^1, ..., c_h^m, d_h\right)$, where d_h represents the "region" of domicile and $\left(c^1, ..., c^m\right)$ are other personal characteristics which can be relevant or irrelevant for discrimination, according to the issue. Thus, for any political issue, there will be a disjoint exhaustive partition of the set $\mathbf{c}$ into two subsets: $\mathbf{c}^R$, which includes all the relevant characteristics, i.e., characteristics that constitutes a legitimate basis for discrimination at regional level; $\mathbf{c}^I$, which includes all the other (irrelevant) characteristics, i.e., characteristics that do not constitute a legitimate basis for discrimination. The sets $\mathbf{c}^R$ and $\mathbf{c}^I$ are such that: (i) $\mathbf{c}^R \subseteq \mathbf{c}$ and $\mathbf{c}^I \subseteq \mathbf{c}$; (ii) $\mathbf{c}^R \cap \mathbf{c}^I = \emptyset$; $\mathbf{c}^R \cup \mathbf{c}^I = \mathbf{c}$. The cardinality and the composition of the sets $\mathbf{c}^R$ and $\mathbf{c}^I$ can be different across issues; moreover, the set of relevant characteristics $\mathbf{c}^R$ could include the domicile d or it could not. Now we introduce the following definition[1].

Definition 2 *We define local issues the spheres of public intervention with respect to which the set of relevant characteristics* $\mathbf{c}^R$ *includes the domicile* d; *we define national issues the spheres of public intervention with respect to which the set of relevant characteristics* $\mathbf{c}^R$ *does not includes the domicile* d.

Notice that this formulation allows for an easy generalization to more than two levels of governments. Say, we could define $\mathbf{c} := \left(c^1, ..., c^m, r, m\right)$ where r and m are the region and the city of domicile, respectively.

Thus, for the local issues, differentiation across regions in the treatment of individuals with equal characteristics is consistent with the horizontal equity principle. Whereas for the national issues differentiation across regions in the treatment of individuals with equal characteristics does violate the HEP.

Two considerations are in order. First, the partition between relevant and irrelevant characteristics is a normative distinction. One component of this partition is the distinction between local and national issues: i.e., the inclusion of the domicile as legitimate base for discrimination. The evaluation of horizontal inequity, in such a framework, is based on the existence of any dissociation between the normative partition and the real, positive, implementation of public policies.

Second, the distinction between local and national issues need not to coincide with the attribution of decisional power at the different levels of government.

[1] In extentions, one may also want to consider issues where competencies across levels of governments are not clearly defined (e.g. overlapping of functions). There may also be some true uncertainty (e.g. legal) about if a particular personal feature belongs to the first or the second type of issue. This could be modelled by employing fuzziness methods. We ignore these aspects here.

There may be cases where ethical principles support an allocation of functions which is different from that observed in reality. For example, suppose we stipulate normatively that the income tax treatment is a national issue; i.e., two individuals who are equals in any relevant characteristics (income, family composition, etc.) should have the same tax burden, irrespective of the region of domicile. If that is the case, and income tax policy is a sphere where both the national and the local governments can intervene, then there will be a source of horizontal inequity whenever the different local governments implement different policies.

A major theoretical point is the following: what is (or should be) the ethical division of labor between local and central governments in providing goods and services to the citizens? A general answer from primary ethical principles is clearly difficult and obviously controversial. However, in practical terms, in most countries, the identification of the relevant characteristics for the different political issues, as well as its partition into local and national issues, is implicitly or explicitly made at a *constitutional level*. As the Constitution of a country represents the legal source for definition of the rights of citizens, it would appear as sensible to consider such a partition (at least in democratic countries) as the natural foundation for an ethical analysis. Hence, the application of the present framework to the evaluation of horizontal inequity requires a preliminary positive exercise in reading the constitutional text.

2 Measuring horizontal inequity: the existing literature

Before proceeding, it is useful to summarizes the general approaches used in the literature to measure horizontal inequity. We refer to horizontal inequity as generated by only one level of government (the focus of the existing literature); successively, we will study the extension of these approaches to the cases of two or more levels of governments. In what follows, for simplicity we assume to be able to quantify individuals' relevant characteristics along a unidimensional scale. The relevant individual "position" could then be represented by money income, by "equivalent income" (using some equivalence scale), by "choice set", and so on.

Let us denote by x this relevant characteristic and let us refer to it, from now on, simply as "income", as most of this literature focuses on the effect of the tax system on income distribution two individuals h and k are equals whenever they have the same income. It is important to notice, however, that in empirical analysis income could be substitute by any observable characteristic or vector of characteristics that define normatively the "equals group".

The income unit can be represented by the individual, the household or the "equivalent adult". It is possible to identify two different approach to the measurement of HI: the *no-reranking* approach and the *classical HI* approach.

2.1 The No-Reranking Approach

In this approach, the HI is identified with the absence of rank reversals (Plotnick, 1981; King, 1985; Shorrocks, 1980). Thus, a tax change that affects the distribution of income between individuals is horizontally equitable if, despite the change, individuals' rankings in the initial distribution are preserved; it is horizontally inequitable if it changes individuals' positions in the distribution. An empirical advantage of this interpretation is that the HI can be measured simply by the number of rank changes.

This approach has been subjected to several criticism. First, the rationale for the identification of HI with rank reversals is rarely spelt out. It could be based on the assumption that the original ranking in the distribution is considered equitable. But how to justify this assumption? The aim of most of the tax-transfer policies whose HI is being assessed is precisely to correct some of the inequity in the original income distribution. In this case, the extent to which rank reversal occur may be an indicator of a policy' s success in equity terms, rather an indicator of its failure. Basing on these considerations, Yitzhaki and Lerman (1995) suppose the final ranks are the equitable ones; therefore, the rank reversal is interpreted as a positive contribution of the tax, which has to be added to the vertical effect to see what is the overall performance of a tax.

Moreover, even considering the pre-tax distribution as equitable, we can observe that for a distribution to be fair, presumable not only would the ordinal ranking be fair but so would be the cardinal dimension of the differences among individuals. Thus, any policy altering the cardinal gaps between incomes would create inequity. But this would not be captured by the definition of HI as no-reranking.

Linked to this reasoning is a further and decisive criticism to this approach: small changes in pre-tax incomes, leading to rank reversal, count as full violation of HE; whereas substantial changes that result in no rank reversal are ignored. So if the pre tax incomes are altered by a non linear function that preserve ranking perfectly, no HI would be observed by using a reranking index. The basic problem is that such a measure does not vary continuously with the magnitude of the effect on each individual (Kaplow, 1989).

2.2 HI as dispersion in post-tax incomes

According to the classical approach, HI is defined as unequal treatment of pre-tax equals. More precisely, one focus upon post-tax inequality among pre-tax equals. Let $S(x)$ denote the group of individuals having exactly x before tax (the "equals group"): there will be HI at the income level x if and only if there is inequality in the after-tax distribution among those having x before tax. That is, HI is manifested as dispersion in ex post incomes of ex ante equal individuals.

Let us consider the group of person having exactly x before tax; suppose now to apply a tax schedule $t(x)$, then the ex post income of people having x ex ante would be:

$$n(x) = x - t(x)$$

But, in case of HI, not all persons having x before tax will have exactly $n(x)$ after tax. Thus, let $n(x,h)$ be the post tax income of household h having x before tax:

$$n(x,h) = n(x) + u(x,h)$$

where the disturbance term $u(x,h)$ capture the unequal treatment of equals introduced by the tax. The tax is locally inequitable (at $S(x)$) unless $u(x,h)$ is degenerated for all h in $S(x)$; it will be globally equitable if and only if $u(x,h)$ is equal to zero for all h and all x.

Hence, in order to measure he overall HI, one needs to choose:

(i) an inequality index to measure the local HI;

(ii) a weighting scheme to aggregate the local indices of HI in an overall HI index.

As a possibility, we could use an *additively decomposable* inequality index. That is, an index such that, for any income distribution X and any partition into subgroups, overall inequality can be expressed as a simple sum of the within and between-group components of inequality. Consider an income distribution $X \in \mathbb{R}_+^N$ with average income μ_X, a partition of X into n subgroups, indexed by $i = 1, ..., n$, and a function $I : \mathbb{R}_+^N \to \mathbb{R}_+$. An additively decomposable inequality index is an index such that, for any income distribution X and any partition into subgroups, the following decomposition is obtained:

$$I = W + B$$

The between groups term B measures inequality in the hipotetycal distribution X_B in which each income in each group $i \in (1, ..., n)$ is replaced by the mean for that group. The within group inequality W measures inequality in the hipotetycal distribution X_W, obtained by re-scaling subgroup incomes so that all the group-means become the same. The within group term W can also be measured as a weighted sum of the inequality values computed for the subgroups:

$$W = \sum_{i=1}^{n} \alpha_i I_i$$

where I_i is the inequality in group i, $\alpha_i = \alpha(p_i, w_i)$ for some function $\alpha : [0,1] \times [0,1] \to \mathbb{R}_+$, and p_i and w_i are respectively the population and the income share of group i. Hence the inequality of income within each sub-group is calculated and then these are summed, using weights of population share, relative incomes or a combination of these two, depending on the particular measure used.

The only measures of relative inequality which satisfy the above decomposition property are the members of the *generalized entropy family.* This is a parametric family of indices E_c defined, when $c \neq 0, 1$ by

$$E_c(X) = \left(\frac{1}{Nc(c-1)}\right)\sum_{i=1}^{N}\left[\left(\frac{x_i}{\mu_X}\right)^c - 1\right]$$

In the special case when $c = 0$, a special measure is obtained, the *mean logarithmic deviation (MLD)*:

$$E_0(X) = \frac{1}{N}\sum_{i=1}^{N}\ln\frac{\mu_X}{x_i}$$

The MLD is the only additively decomposable index for which the subgroup weight are independent of income shares.

Thus, for each group of pre-tax equals $S(x)$, the MLD of post-tax income, call it $I_{S(x)}$, indicates the "local" HI caused by the tax at x. Following the Musgrave' command (1990) to obtain an index of HI which is global and thus comparable with a global index of vertical performance of the tax system, Lambert (1995) suggests a "pure" weighting scheme (income' s independent) in order to obtain an index of the overall HI effect. Thus, we decompose the overall after tax inequality, call it $I(\mathbf{x}-\mathbf{t})$, into two contributions:

(i) a weighted sum of the within equals groups inequality, where for each equals group $S(x)$ we use as weight the population shares $p_{S(x)} = \frac{N_{S(x)}}{N}$:

$$HI = \sum_{S(x)} p_{S(x)} I_{S(x)}$$

which is our global index of HI;

(ii) a measure of the inequality between groups, call it $I^*(\mathbf{x}-\mathbf{t})$, which represents "the post-tax inequality as if a perfectly equitable equal-yield schedule had been applied and not the actual system".

Hence we have

$$I(\mathbf{x}-\mathbf{t}) = HI + I^*(\mathbf{x}-\mathbf{t})$$

Subtracting from the inequality of income before tax, $I(\mathbf{x})$, we obtain

$$I(\mathbf{x}) - I(\mathbf{x}-\mathbf{t}) = I(\mathbf{x}) - I^*(\mathbf{x}-\mathbf{t}) - HI$$

Now, $I(\mathbf{x}) - I^*(\mathbf{x}-\mathbf{t})$ is a measure of the vertical redistribution (VR) of the tax, measured itself by the MLD; $I(\mathbf{x}) - I(\mathbf{x}-\mathbf{t})$ is a measure of the overall redistributive effect (RE). Hence one obtains the decomposition (Lambert and Ramos, 1997):

$$RE = VR - HI$$

according to which HI is identified with loss of vertical performance. To put it differently, the HI index quantifies a gain in performance which could come from eliminating HI within equals groups; this process of elimination would involve no change in tax revenue.

3 Measuring HI in a regional context

In this section we aim at identifying and measuring horizontal inequity in a context where there are at least two levels of government. Generally, there is horizontal inequity whenever equals are treated unequally. To simplify, we keep the assumption that the only personal characteristic which is relevant for discrimination is income; in this scenario, however, also the domicile can play a role.

We have to distinguish between:

- local and national issues;
- local and national policy instruments.

As for the first distinction, are equals in the national issues those with equal incomes; are equals in the local issues those with equal incomes and equal domicile.

As for the second distinction, we now assume that the fiscal treatment, for both the national and local issues, is composed of two distinct and independent instruments: a national income tax t, and local income taxes τ^i, with $i = 1, ..., n$ to indicate the region. However, it will be shown that the approach is suitable to consider public expenditure instead of taxation.

We want to measure the HI in both the national and the local issues. We assume, for the moment, that the HI experienced in the different issues are independent among them. Hence measuring HI in the different issues are independent exercises, and in each exercise one needs to consider the two levels of public intervention, local and national.

3.1 HI and discrimination: a decomposition

A first problem we address amount at distinguishing between differential treatment due to region specific factors and discrimination due to other factors. In this first scenario, no distinction is made between local and national taxes, or more generally, local and national public intervention. A recent model of Ramos and Lambert (2003) may be appropriate here, which extends the HI measurement framework summarized in the previous sections by admitting some deserving attributes into the idealized tax function, in addition to income.

The application of the model proposed by Ramos and Lambert (2003) to the current context goes as follows.

Let $x \in X$ and $d \in D = \{d_1, ..., d_n\}$ denote, respectively, the income level and the domicile. Let u be any undeserving personal attribute and ϵ an error term. In this section we do not distinguish between local and national tax instruments. Hence the tax liability as well as the post tax income are referred to the entire tax system, without reference to the source. We introduce the following notation:

- $t(x, d, u) + \epsilon$. is the total actual tax paid by an individual with characteristics (x, d, u)

- $\tau(x,d)$ is the average tax paid by the group of individuals with characteristics (x,d), i.e., the average tax paid within the equals group $S(x,d)$
- $T(x)$ is the average tax paid by the group of individuals with income x, i.e., the average tax paid within the equals group $S(x) = \cup_{d\in D} S(x,d)$

The total effect of the tax system can be decomposed into three stages:

1. $x \to [x - T(x)]$: at this stage there is not differential treatment among the group $S(x)$, neither on the basis of u, nor on the basis of d. Call $\hat{I}^{post}$ the inequality in the hypotetical distribution obtained in stage 1:

$$VR_T = I^{pre} - \hat{I}^{post}$$

   is the vertical redistribution that would be obtained in absence of any differential treatment based on factors other than x;

2. $[x - T(x)] \to [x - \tau(x,d)]$: at this stage there is differentiation according to the domicile; however, within each equal group $S(x,d)$, there is equal treatment. At each income level x, apply the average tax $\tau(x,d)$ to all individuals in group $S(x,d)$, so obtaining the hypotetical distribution

$$\left((x - \tau(x,d_1))\,\mathbf{1}^{|S(x,d_1)|}, ..., (x - \tau(x,d_1))\,\mathbf{1}^{|S(x,d_1)|}\right)$$

   Let $I_{x-\tau}$ be the inequality in the distribution above: $I_{x-\tau}$ captures the differential treatment based on d which takes place at income x. Considering now all the income levels, we obtain

$$D_\tau = \sum_{x\in X} p_x I_{x-\tau}$$

   which captures the differential treatment based on d which takes place in the entire distribution.

3. $[x - \tau(x,d)] \to [x - t(x,d,u,\epsilon)]$: at this stage, within each equal group $S(x,d)$, there is differentiation according to the attributes u and the error term ϵ. At each income level x, at each equal group $S(x,d)$, there will be inequality of treatment given by I_{x-t}. If, at a given income x, we aggregate across domiciles $\{d_1, ..., d_n\}$, we obtain

$$\sum_{d\in D} p_{x,d} I_{x-t}$$

   which captures the overall differential treatment which takes place at income x. To isolate the differential treatment due to attributes u and ϵ, which takes place at income x, we calculate

$$\sum_{d\in D} p_{x,d}\left(I_{x-t} - I_{x-\tau}\right)$$

Finally aggregating across income levels:

$$H = \sum_{x \in X} p_x \sum_{d \in D} p_{x,d} \left(I_{x-t} - I_{x-\tau} \right)$$

which is the differential treatment due to attributes u and ϵ which takes place in the entire distribution.

Ramos and Lambert (2003) show (Theorem 1) that, in the case of the MLD as inequality index, the following decomposition is obtained:

$$RE = VR_T - D_\tau - H$$

where $RE = (I^{pre} - I^{post})$ is the overall redistributive effect of the entire tax system, VR_T is the redistributive effect of the tax on average (in our case, on the real incomes of individuals), D_τ denotes the loss of VR_T due to the region specific factors in the tax system, and H is the loss from non-domicile related differences in tax treatment apparent in the data (such as undeserving attributes or assessment and recording errors) at given income levels. Moreover Ramos and Lambert (2003) show that, in the case of the MLD,

$$H = \sum_{x \in X} p_x \sum_{d \in D} p_{x,d} I_{x-T}^{post}$$

where I_{x-T}^{post} is post tax inequality among the members of the equal group $S(x,d)$.

Hence, suppose we apply such a model and, for each issue, we isolate: (i) the redistributive effect VR, (ii) the horizontal inequity as differential treatment according to the domicile D_τ, (iii) the horizontal inequity as differential treatment due to other factors H. Then, for the national issues both D_τ and H would count as violation of the HEP; while for the local issues only H would signal the presence of Horizontal inequity.

This model allows one to distinguish between HI due to region specific factors and HI due to other factors; however, it does not allow to disentangle the differential sources of HI, i.e., the regional or the national taxes or benefits. To this issue we turn now our attention. We begin by discussing the problem of measuring HI in the national issues in a context where there are two policy instruments.

3.2 Measuring HI in the national issues

We expand the model used in the existing literature by introducing explicitly two levels of public intervention. We are denoting by x the pre-tax income. Now we consider a national government that imposes a tax $t()$, and regional governments $i = 1, ..., n$ that impose taxes $\left(\tau^1, ..., \tau^n\right)$. The total tax burden of an individual with income x in region i will be

$$T(x) = t(x) + \tau^i(x)$$

and her net income will be

$$n(x) = x - t(x) - \tau^i(x)$$

We can now formalize the requirement of horizontal equity in the national issues.

Axiom 3 *Horizontal Equity in National Issues (HENI). For all individuals h and k, leaving respectively in regions i and j, if $x_h = x_k$ then*

$$(i)\ \ t(x_h) = t(x_k)\ \ \text{and}\ \ (ii)\ \ \tau^i(x_h) = \tau^j(x_k)$$

Clearly, any income tax satisfying HENI will be such that, if $x_h = x_k$ then

$$n(x_h) = n(x_k)$$

We can now propose two different definitions of HI:

$$\begin{aligned} (i)\ \ n(x_h) &\neq n(x_k) \\ (ii)\ \ t(x_h) &\neq t(x_k)\ \text{ or } \tau^i(x_h) \neq \tau^j(x_k) \end{aligned}$$

In the former case, HI is defined as post-tax income inequality among pre-tax equals; in the latter case, HI is defined as inequality in either the national or the regional tax yields among pre-tax equals.

Remark 4 *Cases (i) and (ii) are distinct. To see this, suppose that $x_h = x_k$, $t(x_h) = t(x_k) + \epsilon$ and $\tau^i(x_h) = \tau^j(x_k) - \epsilon$. In this case condition (i) will be satisfied while condition (ii) is violated. More generally, suppose there are two regions, A and B, and two individuals h and k with incomes x and y and leaving respectively in region A and region B. We have:*

$$\begin{aligned} n(x) &= x - t(x) - \tau^A(x) \\ n(y) &= y - t(y) - \tau^B(y) \end{aligned}$$

Let

$$\begin{aligned} t(x) &= t(y) + \epsilon^N \\ \tau^B(y) &= \tau^A(x) + \epsilon^R \end{aligned}$$

Therefore,

$$n(y) - n(x) = y - x + \left(\epsilon^N + \epsilon^R\right)$$

Suppose now $x = y$. If HI is defined as post-tax income inequality among pre-tax equals there will be Horizontal Equity whenever the sum of the disturbance terms ϵ^N and ϵ^R is zero: (i) $\epsilon^N + \epsilon^R = 0$. If HI is defined as inequality in either the national or the regional tax yields among pre-tax equals, there will be Horizontal Equity if and only if both disturbance terms ϵ^N and ϵ^R are equal to zero: (ii) $\epsilon^N = \epsilon^R = 0$. Clearly, condition (ii) implies condition (i), but the contrary is not true. In general, there could be compensations between the national and *regional levels of public intervention that would not be detected by dispersion in the post-tax incomes. The net effect of the two levels of taxation, as captured by the differences in the net incomes $n(x)$ and $n(y)$, need not to be a summation of the differential treatment at the two levels of government.*

Remark 5 *There could be an interaction among the sources of horizontal inequity. Suppose the regional tax τ^j is defined as $\tau^j(x_h, t(x_h))$. In this case, the differential treatment of equals at a national level will manifest itself in both the national and the regional tax burden. To see this, suppose $\tau^j(x_h) = \tau^j t(x_h)$ and consider two individuals h and k such that $x_h = x_k$. Assume there is a horizontal inequitable national tax: $t(x_h) \neq t(x_k)$. In this case, even if we assume a common regional tax rate τ, we obtain that $\tau^j(x_h) = \tau t(x_h)$ and $\tau^j(x_k) = \tau t(x_k)$; as $t(x_h) \neq t(x_k)$ we will have $\tau^j(x_h) \neq \tau^j(x_k)$. The impact of this joint effect on the overall measure of HI is still to be explored.*

Consider the group of individuals having exactly x before tax (the "equals group" $S(x)$). This will be represented by a vector $\left(x\mathbf{1}^{|S(x)|}\right)$, where $\mathbf{1}^{|S(x)|}$ is the unit vector of length $|S(x)|$, the cardinality of $S(x)$; the vectors $\mathbf{t}(x) \in \mathcal{R}^{|S(x)|}$, $\boldsymbol{\tau}(x) \in \mathcal{R}^{|S(x)|}$, and $\mathbf{n}(x) \in \mathcal{R}^{|S(x)|}$, are, respectively, the vectors of national tax, local tax and net incomes corresponding to the equals group $S(x)$. Choose an inequality measure $I : \mathcal{R}^{|S(x)|} \rightarrow \mathcal{R}+$. A "local" measure of HI at $S(x)$ will focus on $I(\mathbf{t}(x))$, $I(\boldsymbol{\tau}(x))$, $I(\mathbf{n}(x))$.

3.2.1 HI as dispersion in post-tax incomes

If we define HI as post-tax income inequality among pre-tax equals, then a local measure of HI will be:

$$LHI_{S(x)} = I(\mathbf{n}(x))$$

We follow Lambert (1995), and choose the MLD as our measure of inequality and a "pure" weighting scheme (income independent) in order to obtain an index of the overall HI effect. Letting $p_{S(x)} = \frac{|S(x)|}{N}$, where N is the overall population size, we have:

$$HI = \sum_{S(x)} p_{S(x)} LHI_{S(x)}.$$

HI is the horizontal inequity jointly introduced by the national and regional tax systems. In this case we could apply Lambert's methodology and obtain the decomposition

$$RE = VR - HI$$

according to which HI is identified with loss of vertical performance. In the current context RE and VR refer to the redistributive effect and the vertical redistribution jointly introduced by the national and regional tax systems, as revealed by the net (post-tax) incomes.

3.2.2 HI as dispersion in tax yields

As an alternative, HI can be defined as inequality in either the national or the regional tax yields among pre-tax equals:

$$\begin{aligned} LHI^N_{S(x)} &= I(\mathbf{t}(x)) \\ LHI^R_{S(x)} &= I(\boldsymbol{\tau}(x)) \end{aligned}$$

where $LHI^N_{S(x)}$ is the local (at $S(x)$) HI introduced by the national tax and $LHI^R_{S(x)}$ is the local (at $S(x)$) HI introduced by the regional tax. This definitions suggests the possibility of decomposing the HI according to the source of discrimination: regional or national.

Also in this case we follow Lambert (1995) in choosing a "pure" weighting scheme in order to obtain an index of the overall HI effect, for both levels of public intervention. Thus we isolate two contributions:

(i) a weighted sum of the within equals groups inequality in national taxes:

$$HI^N = \sum_{S(x)} p_{S(x)} LHI^N_{S(x)}$$

which is our global index of HI introduced by national tax;

(i) a weighted sum of the within equals groups inequality in regional taxes,:

$$HI^R = \sum_{S(x)} p_{S(x)} LHI^R_{S(x)}$$

which is our global index of HI introduced by regional tax[2].

[2] An alternative way to measure the HI generated by the regional taxes would be the following. Let $n^N(x) = x - t(x)$ and $n(x) = n^N(x) - \tau^i(x) = x - t(x) - \tau^i(x)$. First, let us consider only the national tax. Following Lambert (1995) we define the local measure of HI as:

$$LHI^N_{S(x)} = I\left(\mathbf{n}^N(x)\right)$$

and the overall HI as

$$HI^N = \sum_{S(x)} p_{S(x)} I\left(\mathbf{n}^N(x)\right).$$

Now we consider also the regional tax, and we have a local measure

$$LHI_{S(x)} = I(\mathbf{n}(x))$$

and the overall measure

$$HI = \sum_{S(x)} p_{S(x)} I(\mathbf{n}(x)).$$

HI is the horizontal inequity jointly introduced by the national and regional tax systems. A solution to measure the HI introduced by the regional tax is by capturing it as a difference of the two effects. Hence, by using an obvius notation,

$$LHI^R_{S(x)} = I(\mathbf{n}(x)) - I\left(\mathbf{n}^N(x)\right)$$

and the overall HI introduced by the regional tax as

$$HI^R = HI - HI^N$$

In general, we have distinct overall indices of HI for the national and regional levels respectively. A simple aggregation of the two indices would give an overall measure of HI:

$$Overall\ Horizontal\ Inequity:\ HI^N + HI^R$$

As suggested in Remark 4, this index need not to be equivalent to the overall HI index computed as inequality in net incomes among pre-tax equals. Moreover, the possibility of expressing the national and regional horizontal inequities as losses of vertical performance of the entire tax system is still to be explored.

3.3 Measuring HI in the local issues

The analysis of HI in the local issues differs from the analysis performed for the national issues for one aspect only: the definition of equals. Are equals in the local issues those individuals with equal income who also leave in the same region. Hence we now formalize the requirement of horizontal equity in the local issues.

Axiom 6 *Horizontal Equity in Local Issues (HELI). For all individuals h and k, leaving in regions i, if $x_h = x_k$ then*

$$(i)\ t(x_h) = t(x_k)\ \text{ and }\ (ii)\ \tau^i(x_h) = \tau^i(x_k)$$

Clearly, any income tax satisfying HELI will be such that, if $x_h = x_k$ then

$$n(x_h) = n(x_k)$$

According to the axiom above, we can propose two different definitions of HI:

$$\begin{array}{rcl} (i)\ n(x_h) & \neq & n(x_k) \\ (ii)\ t(x_h) & \neq & t(x_k)\ \text{ or } \tau^i(x_h) \neq \tau^i(x_k) \end{array}$$

The application of different regional taxes τ^i τ^j to individuals with the same income x but leaving respectively in regions i and j will not produce any HI. Notice, however, that we are assuming that for the local issues, also the national taxes t can legitimately vary across regions; i.e., the application of different national taxes to individuals with the same income x but different domicile does not count as violation of the horizontal equity principle.

An alternative formulation would require the national tax to be homogeneous in the entire country, while recognizing as equitable different regional tax schedules. To express this view, we need to reformulate the axiom HELI by adding the requirement of equal national tax treatment of individuals with equal incomes, whatever the domicile:

This approach, however, is problematic. The compensation effect explained in Remark 4 suggests that the two terms of regional and national HI could be non additive, therefore suggesting some difficulties in the interpretation of the term HI^R above.

Axiom 7 *Strong Horizontal Equity in Local Issues (SHELI). For all individuals h and k, leaving respectively in regions i and j :*

$$\begin{aligned} (i)\ if\ x_h &= x_k\ then\ t(x_h) = t(x_k) \\ (ii)\ if\ i &= j\ and\ x_h = x_k\ then\ \tau^i(x_h) = \tau^i(x_k) \end{aligned}$$

Clearly, $SHELI$ implies $HELI$: any tax system satisfying axiom $SHELI$ will also satisfy axiom $HELI$.

Hence, according to axiom $SHELI$, the definition of the equals group varies for the regional and local taxes. Therefore, the measurement of HI will have to focus on the dispersion of regional and national taxes among the relevant (and different) groups of equals.

As for the measurement of HI, the approach sketched in the previous section is also valid for the local issues, hence we do not repeat it here.

4 The net fiscal system and HI

In the previous sections we concentrated on the measurement of HI introduced by a tax system. However, the framework can be easily extended to a system where the public intervention involves both tax and benefits. To begin with, consider only the national level, and let $b(x)$ be a positive transfer obtained by an individual with income x. If $t(x)$ is the tax schedule, the net tax will be then

$$t_N(x) = t(x) - b(x)$$

and the net income will be

$$n(x) = x - t(x) + b(x)$$

To extend this model to two levels of government, consider now a region i which imposes a tax τ^i and attributes a benefit β^i. The net regional tax imposed on an individual with income x will be

$$\tau_N^i(x) = \tau^i(x) - \beta^i(x)$$

Consider a person with income x and domicile in region i. Her net tax will be then

$$t_N(x) = t(x) + \tau^i(x) - b(x) - \beta^i(x)$$

and the net income will be

$$n_N(x) = x - t(x) - \tau^i(x) + b(x) + \beta^i(x)$$

Remark 8 *To consider a framework where only public expenditure is involved (i.e., the system is composed of attributed benefits only), it will suffice to assume that $t(x) = \tau^i(x) = 0$ for all x and for all i.*

Let us develop the analysis for the national issues. The extension to local issues is straightforward.

We now formalize the requirement of horizontal equity for a net fiscal system in the national issues.

Axiom 9 *Horizontal Equity for Net fiscal system in National Issues (HENNI). For all individuals h and k, leaving respectively in regions i and j, if $x_h = x_k$ then*

$$(i)\ \ t_N(x_h) = t_N(x_k) \ \ and\ \ (ii)\ \ \tau_N^i(x_h) = \tau_N^j(x_k)$$

Clearly, any income tax satisfying HENNI will be such that, if $x_h = x_k$ then

$$n_N(x_h) = n_N(x_k)$$

In analogy with the previous analysis, we can propose two different definitions of HI:

$$\begin{array}{rcl} (i)\ \ n_N(x_h) & \neq & n_N(x_k) \\ (ii)\ \ t_N(x_h) & \neq & t_N(x_k) \text{ or } \tau_N^i(x_h) \neq \tau_N^j(x_k) \end{array}$$

In the former case, HI is defined as net income inequality among ex-ante equals; in the latter case, HI is defined as inequality in either the national or the regional net tax yields among ex-ante equals.

Consider the group of individuals having exactly x before tax (the "equals group" $S(x)$). This will be represented by the vector $\left(x\mathbf{1}^{|S(x)|}\right)$; the vectors $\mathbf{t}_N(x) \in \mathcal{R}^{|S(x)|}$, $\boldsymbol{\tau}_N(x) \in \mathcal{R}^{|S(x)|}$, and $\mathbf{n}_N(x) \in \mathcal{R}^{|S(x)|}$, are, respectively, the vectors of net national tax, net regional tax and net incomes corresponding to the equals group $S(x)$. Choose an inequality measure $I : \mathcal{R}^{|S(x)|} \to \mathcal{R}+$. A "local" measure of HI at $S(x)$ could focus on $I(\mathbf{t}_N(x))$, $I(\boldsymbol{\tau}_N(x))$, $I(\mathbf{n}_N(x))$.

We will start by studying HI as inequality in net incomes among ex-ante equals.

4.1 HI as dispersion in net incomes

If we define HI as net income inequality among ex-ante equals, then a local measure of HI will be:

$$LHI_{S(x)}^N = I(\mathbf{n}_N(x))$$

Again, we choose the MLD as inequality measure and a "pure" weighting scheme in order to obtain an index of the overall HI effect:

$$HI_N = \sum_{S(x)} p_{S(x)} LHI_{S(x)}^N.$$

In this case Lambert's methodology puts at our disposal the following decomposition of net fiscal incidence progressivity into vertical and horizontal components:

$$RE_N = VR_N - HI_N$$

according to which HI is identified with loss of vertical performance. In the current context RE_N and VR_N refer to the redistributive effect and the vertical redistribution jointly introduced by the national and regional net fiscal systems, as revealed by the net incomes.

4.2 HI as dispersion in net tax yields

The analysis of HI as inequality in net tax treatment appears to be problematic, as the net individual incidence could be either positive or negative. As a consequence, the analytical results of the existing literature cannot be easily extended to this context.

However, let us assume we are in a framework where only public expenditure is involved (i.e., the system is composed of attributed benefits only). That is, we assume that $t(x) = \tau^i(x) = 0$ for all x and for all i. Consider a person with income x and domicile in i. Her net regional tax will be

$$\tau_N^i(x) = \beta^i(x)$$

Her net national tax

$$t_N(x) = b(x)$$

and her net income will be

$$n_N(x) = x + b(x) + \beta^i(x)$$

In this case the vectors $\mathbf{t}_N(x) \in \mathcal{R}$ and $\boldsymbol{\tau}_N(x) \in \mathcal{R}^{|S(x)|}$ are positive and we can extend the methodology developed in the previous sections.

Thus, HI is now defined as inequality in either the national or the regional attributed benefits among ex-ante equals:

$$\begin{aligned} LHI_{S(x)}^N &= I(\mathbf{t}_N(x)) \\ LHI_{S(x)}^R &= I(\boldsymbol{\tau}_N(x)) \end{aligned}$$

where $LHI_{S(x)}^N$ is the local (at $S(x)$) HI introduced by the national system and $LHI_{S(x)}^R$ is the local (at $S(x)$) HI introduced by the regional system. Also in this case we choose a "pure" weighting scheme to obtain an index of the overall HI effect, for both levels of public intervention. Thus we isolate two contributions:

(i) a weighted sum of the within equals groups inequality in national benefits:

$$HI^N = \sum_{S(x)} p_{S(x)} LHI_{S(x)}^N$$

which is our global index of HI introduced by national system;

(i) a weighted sum of the within equals groups inequality in regional benefits,:

$$HI^R = \sum_{S(x)} p_{S(x)} LHI^R_{S(x)}$$

which is our global index of HI introduced by regional system.

In general, we have distinct overall indices of HI for the national and regional levels respectively. A simple aggregation of the two indices would give an overall measure of HI:

$$Overall\ Horizontal\ Inequity: \ HI^N + HI^R$$

This index need not to be equivalent to the overall HI index computed as inequality in net incomes among ex-ante equals.

4.3 An extension

As far as the national issues is concerned, there is HI whenever two individuals with the same relevant characteristics x are treated differently, whatever their domicile. Hence, considering the group of equals $S(x)$, the HI experienced by this group will be measured as dispersion in the distribution of benefits received by the components of $S(x)$. If all the individuals in $S(x)$ receive the same amount of benefits, there is no HI; otherwise, there is HI. Let $\mathbf{t}(x)$ be the vector of benefits received by the members of $S(x)$.

Local HI at x will be

$$LHI(x) = I(\mathbf{t}(x))$$

Now we partition the set $S(x)$according to the regions, so obtaining $\left(S^1(x), ..., S^n(x)\right)$, where:

- $S^i(x)$ is the set of individuals with characteristics s and domicile in region i;

- $S^i(x) \cap S^i(x) = \varnothing$ for all $i, j = 1, ...n$;

- $\cup_{i=1}^n S^i(x) = S(x)$.

Let $p_i(x)$ be the population share of group $S(x)$ in region i. Let $\mathbf{t}_i(x)$ the distribution of benefits attributed to members of $S_i(x)$, so that $\mathbf{t}(x) = \sum_{i=1}^n p_i(x)\mathbf{t}_i(x)$. Therefore $I\ (\mathbf{t}_i(x))$ will be the HI experienced in region i, and

$$LHI^W(x) = \sum_{i=1}^n p_i(x) I(\mathbf{t}_i(x))$$

is the overall HI experienced within the regions.

Now, let $\mu_i^t(x)$ be the average benefit received by individuals with characteristics x and domicile in region i, and let $\mathbf{1}^{N_i}$ the unit vector of length N_i. The distribution

$$\mathbf{t}^B(x) = \left(\mu_1^t(x)\mathbf{1}^{N_1}, ..., \mu_n^t(x)\mathbf{1}^{N_n}\right)$$

eliminates all inequality within regions. Hence

$$LHI^B(x) = I\left(\mathbf{t}^B(x)\right)$$

measures the violation of the horizontal equity principle that is to be attributed to differential treatment across regions.

In the case of the MLD, by making use of Lambert's decomposition, we obtain

$$I\left(\mathbf{t}^B(x)\right) = I(\mathbf{t}(x)) - \sum_{i=1}^{n} p_i I(\mathbf{t}_i(x))$$

that is,

$$LHI^B(x) = LHI(x) - LHI^W(x)$$

Aggregating across x, hence aggregating across the groups of equals, we obtain $HI^B = \sum_x LHI^B(x)$ and therefore

$$HI^B = HI - HI^W$$

Consider that, often, with aggregate data, we cannot observe the distributions $\mathbf{t}(x)$ and $\mathbf{t}_i(x)$ for $i = 1, ..., n$; but we can observe $\mathbf{t}^B(x)$ at all x. Hence we can calculate HI^B.

References

[1] ARONSON R., P. JOHNSON and P.J. LAMBERT (1994). Redistributive effect and unequal income tax treatment. Economic Journal, vol 104, pp. 262-270.

[2] ATKINSON A.B. (1980). Horizontal equity and the distribution of the tax burden. In H.AARON and M.BOSKIN (eds.) The Economics of Taxation, Washington DC: Brookings Institution.

[3] BERLIANT M.C. and R.P. STRAUSS (1985). The horizontal and vertical equity characteristics of the federal individual income tax, 1966-1977. In M.DAVID and T.SMEEDING (eds.) Horizontal Equity, Uncertainty and Economic Well-Being, NBER Studies in Income and Wealth, vol. 50, Chicago: University Press.

[4] CUBEL M and LAMBERT P.J (2002) A regional approach to income tax policy. Public Finance Review, vol 30, pp. 124-143.

[5] DUCLOS J-Y and LAMBERT P (2000) A normative and statistical approach to measuring classical horizontal inequity. Canadian Journal of Economics, vol 33, pp. 87113.

[6] JENKINS S.P. (1988). Empirical measurement of horizontal inequity. Journal of Public Economics, vol. 37,pp.305-329.

[7] JENKINS S.P. and P.J. LAMBERT (1996). Horizontal inequity measurement: a basic reassessment. In SILBER J. (Ed.) Income Inequality Measurement : From theory to Pratice. Norwell, MA: Kluwer Academic Publishing.

[8] KAPLOW, L. (1989). Horizontal inequity: measures in search of a principle. National Tax Journal, vol. 42, pp. 139-154.

[9] KING M. A. (1983). An index of inequality: with application to horizontal equity and social mobility. Econometrica, vol. 51, pp. 99-115.

[10] LAMBERT (1993) Inequality reduction through the income tax. Economica 60, 357-365

[11] LAMBERT P.J. (2002). The Distribution and Redistribution of Income (3rd edition). Manchester: University Press.

[12] LAMBERT P.J. (1995). On the measurement of horizontal inequity. Working Paper WP/95/135. Washington DC: World Bank.

[13] LAMBERT P.J. and X. RAMOS (1997). Vertical redistribution and horizontal inequity. International Tax and Public Finance.

[14] LERMAN R. and S.YITZHAKI (1995a). Changing ranks and the inequality impacts of taxes and transfers. National Tax Journal, vol. 48, pp. 45-59.

[15] LAMBERT and YITZHAKI (1995b) Equity, equality and welfare. European Economic Review, 39, 674-682.

[16] MUSGRAVE R.A. (1976). ET, OT and SBT. Journal of Public Economics, vol.6, pp. 3-16.

[17] MUSGRAVE R.A. (1983). The nature of horizontal equity and the principle of broad-based taxation: a friendly critique, in J.C. HIND (ed.) Taxation issues of the 1980s (Australian Tax Foundation).

[18] MUSGRAVE R.A. (1990). Horizontal equity, once more. National Tax Journal, vol. 43, pp. 113-122.

[19] PERAGINE V. Opportunity egalitarianism and income inequality: the rank-dependent approach. *Math. Soc. Sciences, (2002),* 44, pp.45-64.

[20] PLOTNICK, R. (1982). The concept and measurement of horizontal inequity. Journal of Public Economics, vol. 17, pp. 373-391.

[21] RAWLS, J. (1971), A Theory of Justice. Harvard University Press, Cambridge.

[22] SEN A. and B.WILLIAMS (eds.) (1983). Utilitarianism and Beyond. Cambridge: Cambridge University Press.

[23] SHORROCKS, A.F. (1984). Inequality decomposition by population subgroups. Econometrica, vol 52, pp. 1369-1385.